Received: *December 05, 2001*, Accepted: *May 23, 2001*

Contact interactions in D-brane models

Ignatios Antoniadis* **and Karim Benakli**†

CERN Theory Division CH-1211
Genève 23, Switzerland
E-mail: ignatios.antoniadis@cern.ch, karim.benakli@cern.ch

Alexandre Laugier

Centre de Physique Théorique, Ecole Polytechnique
91128 Palaiseau, France
(Unité Mixte du CNRS et de l'EP, UMR 7644)
E-mail: alexandre.laugier@cpht.polytechnique.fr

Abstract: We compute the tree-level four-point scattering amplitudes in string models where matter fields live on D-brane intersections. Extracting the contribution of massless modes, we are left with dimension-six four-fermion operators which in general receive contributions from three different sources: exchange of massive Kaluza-Klein excitations, winding modes and string oscillator states. We compute their coefficients and extract new bounds on the string scale in the brane-world scenario. This is contrasted with the situation where matter fields arise from open strings with both ends confined on the same collection of D-branes, in which case the exchange of massive string modes leads to dimension-eight operators that have been studied in the past. When matter fields live on brane intersections, the presence of dimension-six operators increases the lower bound on the string scale to 2–3 TeV, independently of the number of large extra dimensions.

Keywords: Extra Large Dimensions, D-branes, Beyond Standard Model, Compactification and String Models.

*On leave from *Centre de Physique Théorique, Ecole Polytechnique, F-91128 Palaiseau*
†Department of Physics, McGill University, Montréal, QC, H3A 2TS, Canada

JHEP05(2001)044

Contents

1. Introduction

The world-volume of a Dp-brane [1] can be described as a $(p+1)$-dimensional space-time where open strings can end and propagate. This free propagation of the string endpoints is described by imposing Neumann (N) boundary conditions for the world-sheet fields. In the remaining $9-p$ transverse dimensions the endpoints of these open strings are confined at the location of the brane and satisfy Dirichlet (D) conditions. The existence of a string world-sheet description for the D-brane dynamics of world-volume and bulk states provides a powerful tool that allows to compute higher derivative corrections as well as string loop corrections.

A generic configuration of D-branes contains branes with world-volumes of different dimensionalities. Along a given direction, open strings stretched between these different branes could satisfy NN, DD, or ND boundary conditions, depending on whether this direction is part of the world-volume of both D-branes, transverse to both of them, or along the world-volume of the one but transverse to the other, respectively.

Orbifold compactifications of type-I strings [2, 3] provide the simplest framework allowing to investigate the possibility that standard model fieldsare identified with degrees of freedom confined on D-branes. Search for such realistic models indicates that the configuration of branes should contain at least two sets, for instance D3- and D7-branes (or a T-dual configuration). While gauge particles are associated to NN or DD open strings with both ends on the same set of branes, chiral matter fields are usually localized on brane intersections and satisfy ND boundary conditions in some internal directions. Besides providing chirality, this choice avoids possible phenomenological problems related to the fact that if all matter fields were associated to NN or DD strings as the gauge fields, the lowest Kaluza-Klein (KK) excitations would be stable. This would lead, in particular, to stable charged states with TeV masses in models with low compactification or string scales [4, 5].

Confining standard model states on D-branes opens new possibilities for phenomenological applications of string theory as it becomes possible to lower its fundamental scale much below the Planck mass [6, 7, 8]. The most spectacular of these proposals are scenaria with the string scale $M_s \equiv l_s^{-1}$ lying at energies as low as a few TeV, where a plethora of new phenomena could be observable at future colliders. In order to derive constraints on these models, it is important to study deviations from the standard model expectations for low-energy cross-sections.

Such deviations manifest themselves for instance as the appearance of higher-dimensional operators. Here we discuss these operators in the case of orbifolds with two types of branes that we take for concreteness as D3- and D7-branes. We then find that in addition to the dimension-six four-fermion operators induced by the exchange of KK states or winding modes, there are also dimension-six operators induced by massive string oscillators. All such contact interactions appear when standard model matter fields are identified with massless modes of open strings stretched between the D3- and D7-branes (37 strings), satisfying ND boundary conditions along four internal directions. These effects dominate the dimension-eight operators [5, 9] obtained for the case where matter fields are identified with massless modes of DD strings (33 or 77 strings).

In the context of the above framework, one can assign two different embeddings of the Standard Model:

1. One possibility is that all observable gauge fields live on the world-volume of D3-branes, while the D7-branes are in general extended (partially) in the bulk. If matter fields come from open strings with both ends on the D3-branes, the presence of D7-branes is irrelevant for our purposes and this case is reduced to

the one studied in ref. [9] and summarized in section 3.1. The only possible source of massive exchanges are string oscillator modes that lead in this case to dimension-eight effective operators.

2. Other possibilities are obtained when the Standard Model is splitted in two factors living correspondingly on the D3- and D7-branes which are transverse simultaneously to the two remaining extra dimensions of the bulk (of mm size) [5, 10]. For instance, strong and weak interactions could be confined on the D3- and D7-branes, respectively, and the size of the four-dimensional internal volume along the D7-branes could account for the weakness of the SU(2) relative to the SU(3) couplings.

As we mentioned above, in both cases, the presence of matter fields originating from 37 strings leads to dimension-six operators which receive contributions from exchanges of KK modes of the 77 strings (having both ends on the D7-branes). In scenario (1), they are associated to new (exotic) degrees of freedom with new interactions to Standard Model states. Since these are model dependent and presumably suppressed in realistic models, they can not be used to obtain model independent experimental bounds. On the other hand, in scenario (2), the KK states correspond to heavy excitations of Standard Model gauge degrees of freedom. The effects of their exchanges should then be used to obtain experimental bounds that have been studied in the past [4, 11, 12].

In addition to the KK excitations of 77 states, there are string winding modes around the D7-brane world-volume compact directions which carry D3-brane quantum numbers and can be exchanged among 37 states. However, their contribution is exponentially suppressed in the large volume limit and can be neglected. The remaining contributions to dimension-six operators come from the exchange of massive oscillators of 33 and 77 strings and can be extracted explicitly. They can be used to derive model independent bounds on the string scale, in the context of our framework.

It is important to note that our results remain valid in non-supersymmetric models with brane supersymmetry breaking, where some D-branes are replaced by anti-D-branes of the same type [13]. Supersymmetry is then broken (to lowest order) on the world-volume of the anti-D-branes.

Our paper is organized as follows. In section 2, we recall the basic properties of D-brane models and define our framework. In section 3, we compute all possible four-fermion tree-level scattering amplitudes, while in section 4, we extract the four-fermion contact interactions. For completeness, we also present the computation of the four-scalar amplitude in appendix A. The reader who is not interested in the detailed derivations can skip sections 3 and 4 and go directly to section 5, which summarizes our results. Finally, in section 6, we perform the numerical analysis and derive the bounds on the string scale.

JHEP05(2001)044

	0	1	2	3	4	5	6	7	8	9	located at
$\mathrm{D}3_I$-branes:	N	N	N	N	D	D	D	D	D	D	$X_{i+3} = a_i^I R_i$
$\mathrm{D}7_1$-branes:	N	N	N	N	D	D	N	N	N	N	$X_4 = X_5 = 0$
$\mathrm{D}7_2$-branes:	N	N	N	N	N	N	D	D	N	N	$X_6 = X_7 = 0$
$\mathrm{D}7_3$-branes:	N	N	N	N	N	N	N	N	D	D	$X_8 = X_9 = 0$

Table 1: Geometrical set-up of the branes.

2. The D-brane model

In this work we consider orbifold compactifications of type-I string theory to four dimensions. Gauge degrees of freedom are described by open strings with ends confined on sets of D-branes. A Dp-brane is obtained by imposing Dirichlet (D) boundary conditions for the endpoints of open strings along the $9-p$ directions transverse to the brane and Neumann (N) conditions along the longitudinal $p+1$ (space + time) directions. Generically, N coincident Dp-branes give rise to a $\mathrm{U}(N)$ gauge group, that can be reduced to the orthogonal or symplectic maximal subgroup by the orientifold projection.

In the supersymmetric case, such a generic vacuum contains four types of Dp-branes which upon appropriate T-dualities can be mapped to one set of D3-branes and three different sets of D7-branes (see table 1), where for simplicity, the six internal coordinates X_{i+3}, $i=1,\ldots,6$ are compactified on a six-dimensional torus with radii R_i. For generality, we have allowed the D3-branes to be localized at different points in the transverse directions parameterized by $a_i^I R_i$ where I labels the branes. Their separations break the gauge group accordingly. Recently, non-supersymmetric models with supersymmetry broken on the branes have been constructed [13]. These models contain the same type of branes with the difference that some of them are replaced by anti-D-branes (having opposite Ramond-Ramond charges). Their numbers are constrained by the cancellation of all Ramond-Ramond charges in the compact internal space, as required by the Gauss law.

The open string spectra contain two kinds of states: gauge fields arise from strings with both ends on the same set of branes, which therefore satisfy the same boundary condition for both ends (NN or DD). Matter fields, on the other hand, can also arise from strings stretched between two different sets of branes. These strings have ND boundary conditions along four of the internal directions. They transform in the bifundamental representation of the two gauge groups associated to the two sets of branes. Moreover, orientation reversal of these strings amounts to complex conjugation. The mass formulae for the various cases are:

- Open strings with one end on the $\mathrm{D}3_I$ and the other on the $\mathrm{D}3_J$-branes:

$$M_{33}^2 = \sum_{i=1}^{6} \frac{(m_i + a_i^{IJ})^2 R_i^2}{l_s^4} + \frac{N}{l_s^2}\,, \tag{2.1}$$

where $a_i^{IJ} = a_i^J - a_i^I$ parameterizes the separation $a_i^{IJ} R_i$ of the branes along the direction i and N is the integer string oscillator number. These states have no KK excitations but have winding modes.

- Open strings with both ends on the same set of D7-branes:

$$M_{77}^2 = \sum_{\perp} \frac{n_\perp^2 R_\perp^2}{l_s^4} + \sum_{\|} \frac{n_\|^2}{R_\|^2} + \frac{N}{l_s^2} . \tag{2.2}$$

These states have KK excitations along the four longitudinal ($\|$) internal dimensions and winding modes along the remaining two transverse ($\perp$) ones.

- Open strings with one end on the D3 and the other on a D7 set of branes:

$$M_{37}^2 = \sum_{\perp} \frac{n_\perp^2 R_\perp^2}{l_s^4} + \frac{N}{2l_s^2} , \tag{2.3}$$

These states have winding modes only along the two directions transverse to both sets of branes. The simultaneous absence of KK and winding modes along the remaining four internal directions associated to ND boundary conditions reflects the property that the endpoints of these strings live in the intersection of D3- and D7-branes.

- Open strings stretched between two different sets of D7-branes:

$$M_{77'}^2 = \sum_{\|} \frac{n_\|^2}{R_\|^2} + \frac{N}{2l_s^2} . \tag{2.4}$$

These states live in the intersection of D7 and D7' and have only KK excitations along the two directions longitudinal to both sets of branes.

Note that the states 77 and 77' are related by T-duality to those of 33 and 37 states, respectively. The open string endpoints carry also gauge indices that are described by Chan-Paton matrices λ.

Besides the open string spectrum, there are closed strings propagating in the whole ten-dimensional space. Their massless modes describe particles with gravitational coupling to matter which include the graviton, dilaton as well as some model dependent states, such as graviphotons and moduli fields. Their masses are given by:

$$M_{\rm closed}^2 = \sum_{i=1}^{6} \frac{m_i^2}{R_i^2} + \sum_{i=1}^{6} \frac{n_i^2 R_i^2}{l_s^4} + \frac{4N}{l_s^2} . \tag{2.5}$$

JHEP05(2001)044

3. Four-fermion scattering amplitudes

In this section, we compute tree-level four-point amplitudes involving massless fermions. Each external state is described by the insertion of a vertex operator $\mathcal{V}^{(a)}$ $a = 1, \ldots, 4$ on the boundary of the world-sheet surface with topology of a disk. Using conformal transformations, the disk can be mapped to the upper-half complex plane and the world-sheet boundary to the real line. There are many possible orderings of the 4 vertex operators along the line at the positions x_i. Using $\mathrm{SL}(2, \mathbb{R})$ conformal invariance one can fix these positions at $0, x, 1, \infty$. The corresponding four-point ordered amplitude is:

$$(2\pi)^4 \delta^{(4)} \left(\sum_a k_a \right) A(1,2,3,4) = \frac{-i}{g_s l_s^4} \int_0^1 dx \Big\langle \mathcal{V}^{(1)}(0, k_1) \mathcal{V}^{(2)}(x, k_2) \mathcal{V}^{(3)}(1, k_3) \times \\ \times \mathcal{V}^{(4)}(\infty, k_4) \Big\rangle \,, \tag{3.1}$$

where k_i are the space-time momenta. The total scattering amplitude $\mathcal{A}_{\text{total}}(1,2,3,4)$ is obtained by summing over all possible orderings of the vertices. Defining $\mathcal{A}(1,2,3,4) = A(1,2,3,4) + A(4,3,2,1)$, one has:

$$\mathcal{A}_{\text{total}}(1,2,3,4) = \mathcal{A}(1,2,3,4) + \mathcal{A}(1,3,2,4) + \mathcal{A}(1,2,4,3) \,. \tag{3.2}$$

These amplitudes depend on kinematical invariants that can be expressed in terms of the Mandelstam variables:

$$s = -(k_1 + k_2)^2 \,, \qquad t = -(k_2 + k_3)^2 \,, \qquad u = -(k_1 + k_3)^2 \,. \tag{3.3}$$

3.1 Four fermions from DD open strings

The simplest and well-known case [9, 14] is the interaction of four massless fermions living on the same set of D3- or D7-branes. The corresponding vertex operator associated with an open string stretched between the coincident branes I and J is given by (in the $-1/2$ ghost picture):

$$\mathcal{V}^{(a)}_{\mathrm{DD}}(x_a, k_a) = \sqrt{2 g_s\, l_s^3}\, \lambda^a_{IJ} u^{(a)}_\alpha S^\alpha_{(10)}\, e^{-\phi/2} e^{i k_a \cdot X}(x_a) \,, \tag{3.4}$$

with $k_a^2 = 0$, λ the corresponding Chan-Paton matrix and $u^{(a)}_\alpha$ the spinor polarizations. Here ϕ is the superconformal ghost field and $S^\alpha_{(10)}$ the spin fields of SO(10) [15]:

$$S^\alpha_{(10)} =: e^{\frac{i}{2} \vec{\tilde{t}}_\alpha \cdot \vec{H}} : \,, \qquad \vec{H} = \{H_1, \ldots, H_5\} \,, \qquad \vec{\tilde{t}}_\alpha = \{\pm, \pm, \pm, \pm, \pm\} \,, \tag{3.5}$$

where H_i are two-dimensional bosonic fields and there is no correlation among the $\pm$ in the different positions. The GSO projection projects half of the components of the spinors $S^\alpha_{(10)}$, keeping for instance those with $\prod_{i=1}^5 \tilde{t}_i = +$. The simultaneous presence of D3- and D7-branes results in further projecting out half of the spinors by an appropriate implementation of Dirichlet boundary conditions (for instance $t_2 t_3 = -$).

In order to introduce also a flavor (family) index let us consider a concrete example of compactification to four dimensions based on the $\mathbb{Z}_3$ orientifold which allows to construct three-family models [3]. The $\mathbb{Z}_3$ orbifold group action transforms the three complex coordinates $z_k = X_{k+3} + iX_{k+4}$ and the world-sheet fields H_i as:

$$\begin{aligned} z_k &\longrightarrow g\, z_k = e^{i2\pi/3} z_k\,, \\ \{H_1, H_2, H_3, H_4, H_5\} &\longrightarrow \left\{H_1, H_2, H_3 + \frac{2\pi}{3}, H_4 + \frac{2\pi}{3}, H_5 + \frac{2\pi}{3}\right\}. \end{aligned} \tag{3.6}$$

The vector $\vec{\vec{t}}_\alpha$ can be decomposed as $\vec{\vec{t}}_\alpha = \{\vec{s}_\alpha, \vec{t}_\alpha\}$ with $\vec{t}_\alpha = \{t_1, t_2, t_3\}$ while $\vec{s}_\alpha = \{s_1, s_2\}$ defines the four-dimensional helicity. For instance left-handed two-component spinors correspond to $\vec{s}_\alpha = \{+,+\}$ and $\{-,-\}$, while right-handed ones are associated with $\vec{s}^{\dot\alpha} = \{+,-\}$ and $\{-,+\}$. Only one left-handed two-component spin field corresponding to $\vec{t} = \{+,+,+\}$ and $\vec{s}_\alpha = \{+,+\}$, $\{-,-\}$ is left invariant by the $\mathbb{Z}_3$ action. It gives rise to the gauginos in four-dimensional $N = 1$ supersymmetric theories. On the other hand, through the tensor product of the remaining spin-fields with Chan-Paton matrices that transform with opposite phases, one obtains three families of matter fermions associated with $\vec{t} = \{+,-,-\}$, $\{-,+,-\}$ and $\{-,-,+\}$.

Using:

$$\langle \phi(z)\phi(w)\rangle = -\ln(z-w)\,, \qquad \langle H_i(z) H_j(w)\rangle = -\delta_{i,j} \ln(z-w) \tag{3.7}$$

a straightforward computation of the ordered amplitude gives:

$$\begin{aligned} \mathcal{A}(1,2,3,4) = -2g_s\, l_s^2\ \mathrm{tr}\left[\lambda^1\lambda^2\lambda^3\lambda^4 + \lambda^4\lambda^3\lambda^2\lambda^1\right] \times \\ \times \int_0^1 dx\, x^{-1-s\,l_s^2}\,(1-x)^{-1-t\,l_s^2}\Big[\bar{u}^{(1)}\gamma_M^{(10)} u^{(2)}\bar{u}^{(4)}\gamma^{(10)M} u^{(3)}(1-x) + \\ + \bar{u}^{(1)}\gamma_M^{(10)} u^{(4)}\bar{u}^{(2)}\gamma^{(10)M} u^{(3)} x\Big]\,, \end{aligned} \tag{3.8}$$

where $\gamma_M^{(10)}$ are the γ-matrices in 10 dimensions. As an example we will consider all four fermions of the same flavor, more precisely two with the same internal helicity $\vec{t}$ and two with the opposite. In this case $\gamma_M^{(10)}$ are reduced to the usual four-dimensional γ-matrices γ^μ. The corresponding ordered amplitudes for various four-dimensional helicities are then given by [9]:

$$\begin{aligned} \mathcal{A}(1_L^-, 2_R^+, 3_L^-, 4_R^+) &= -4\, g_s\ \mathrm{tr}\left[\lambda^1\lambda^2\lambda^3\lambda^4 + \lambda^4\lambda^3\lambda^2\lambda^1\right] \cdot \frac{u^2}{st}\mathcal{F}(s,t)\,, \\ \mathcal{A}(1_L^-, 2_R^+, 3_R^-, 4_L^+) &= -4\, g_s\ \mathrm{tr}\left[\lambda^1\lambda^2\lambda^3\lambda^4 + \lambda^4\lambda^3\lambda^2\lambda^1\right] \cdot \frac{t}{s}\mathcal{F}(s,t)\,, \\ \mathcal{A}(1_L^-, 2_L^+, 3_L^-, 4_L^+) &= -4\, g_s\ \mathrm{tr}\left[\lambda^1\lambda^2\lambda^3\lambda^4 + \lambda^4\lambda^3\lambda^2\lambda^1\right] \cdot \frac{s}{t}\mathcal{F}(s,t)\,, \end{aligned} \tag{3.9}$$

while the other non-vanishing processes can be trivially obtained by parity reflection. All the amplitudes above are expressed in terms of the effective field theory result multiplied by the string form-factor [16]

$$\mathcal{F}(s,t) = \frac{\Gamma(1-l_s^2 s)\Gamma(1-l_s^2 t)}{\Gamma(1-l_s^2 s - l_s^2 t)}\,. \tag{3.10}$$

In the low energy limit $|s\, l_s^2| \ll 1$, $|t\, l_s^2| \ll 1$, it can be expanded as:

$$\mathcal{F}(s,t) = 1 - \frac{\pi^2}{6}\frac{st}{M_s^4} + \cdots\,. \tag{3.11}$$

Note that the first correction to the field theory result, obtained from the term proportional to st in eq. (3.11), corresponds to dimension-eight effective operators.

The above result remains valid in non-supersymmetric models where the fermions arise from open strings ending on the same set of anti-branes.

3.2 Four ND fermions from two sets of branes

The vertex operator describing the emission of a massless fermion originating from an open string stretched between the D7-brane j and the D3-brane I is given by:

$$\mathcal{V}_{\rm ND}^{(a)}(x_a,k_a) = 2^{1/4}\sqrt{g_s\, l_s^3}\,\lambda_{j_a I_a} u_\alpha^{(a)} S_{(6)}^\alpha \prod_{i=1}^{4} \sigma_\pm^i\, e^{-\phi/2} e^{ik_a\cdot X}(x_a)\,, \tag{3.12}$$

where $\sigma_\pm^i$ is the Z_2-twist operator acting on the direction i, with conformal dimension $1/16$; σ_+ changes a Dirichlet to Neumann boundary condition transforming a Dp- to a D$(p+1)$-brane while the anti-twist σ_- does the reverse, transforming a D$(p+1)$- to a Dp-brane. Each ND vertex operator contains four such twist fields. $S_{(6)}^\alpha$ are the spin fields of SO(6):

$$S_{(6)}^\alpha =: e^{\frac{i}{2}\vec{t}_\alpha \vec{H}} :\,, \qquad \vec{H} = \{H_1, H_2, H_3\}\,, \qquad \vec{t}_\alpha = \{\pm,\pm,\pm\}\,. \tag{3.13}$$

The GSO projection projects half of the components of the spinors $S_{(6)}^\alpha$ keeping for instance those with $\prod_{i=1}^3 t_i = +$. Here $\vec{t}_\alpha \equiv \{\vec{s}_\alpha, s_1 s_2\}$ with $\vec{s}_\alpha = \{s_1, s_2\}$ giving the four-dimensional helicity as for the DD case. The Chan-Paton matrices λ transform in the bifundamental representation of the D3 and D7 gauge groups, in the simplest case $U(N_3)\times U(N_7)$, respectively, and can be represented by $(N_3+N_7)\times(N_3+N_7)$ matrices with one non-vanishing off-diagonal element in a complex basis.

Let us first consider the amplitude involving four ND fermions from open strings stretched between two different sets of branes which we choose to be D3 and D7 of the third type in table 1, transverse to the 8th and 9th directions. A non-vanishing result requires at most two sets of coincident D3-branes I and J. The non-trivial part of the computation involves the correlation function of two pairs of twist-anti-twist operators [4, 17, 18]:

$$\langle\sigma_+(0)\sigma_-(x)\sigma_+(1)\sigma_-(\infty)\rangle = \frac{[x(1-x)]^{-1/8}}{[F(x)]^{1/2}} \sum_{n_i\in\mathbb{Z}} e^{-\pi\tau\sum_{i=1}^4 (n_i+a_i^{IJ})^2 R_i^2\, l_s^{-2}}\,, \tag{3.14}$$

where $F(x) = F(1/2, 1/2; 1; x)$ is the hypergeometric function

$$\begin{aligned} F(x) &\equiv \int_0^1 dy\, y^{-1/2}\,(1-y^{-1/2})\,(1-xy)^{-1/2}\,, \qquad \tau(x) \equiv \frac{F(1-x)}{F(x)}\,, \\ x &\equiv \frac{\theta_2^4}{\theta_3^4}(i\tau(x))\,, \end{aligned} \tag{3.15}$$

with θ's the Jacobi theta-functions [19].

Putting together all correlation functions, the ordered four-point amplitude is given by:

$$\begin{aligned} \mathcal{A}(1_{j_1I_1}, 2_{j_2I_2}, 3_{j_3I_3}, 4_{j_4I_4}) = -g_s l_s^2 &\int_0^1 dx\, x^{-1-s\,l_s^2}\,(1-x)^{-1-t\,l_s^2}\,\frac{1}{[F(x)]^2}\,\times \\ &\times \left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}(1-x) + \bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)} x\right]\times \\ &\times \left\{ \frac{\delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3}}{l_s^{-4}\prod_{i=1}^4 R_i} \sum_{m_i\in\mathbb{Z}} e^{2i\pi\sum_{i=1}^4 m_i a_i^{I_1I_3}}\, e^{-\pi\tau\sum_i \frac{m_i^2 l_s^2}{R_i^2}} + \right. \\ &\left. + \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3} \sum_{n_i\in\mathbb{Z}} e^{-\pi\tau\sum_{i=1}^4 (n_i+a_i^{I_1I_3})^2 R_i^2\, l_s^{-2}} \right\}, \end{aligned} \tag{3.16}$$

where we made explicit the dependence on Chan-Paton indices. The total amplitude for the scattering of four fermions is obtained by summing over different orderings. Note that the γ-matrices appearing in the second line of (3.16) should be in general the six-dimensional ones. However, as in the previous case of DD fermions we have chosen the internal fermion polarizations such that only the four-dimensional matrices appear.

The field theory result is obtained by taking the limit of coincident vertices $x \to 0$ or $x \to 1$. The behavior of the special function $F(x)$ in these limits is given by:

$$\begin{aligned} &x \longrightarrow 0\ : \qquad F(x) \sim 1\,, \qquad F(1-x) \sim \frac{1}{\pi}\ln\frac{\delta}{x}\,, \qquad \tau \longrightarrow \infty\,, \\ &x \longrightarrow 1\,, \qquad F(x) \sim \frac{1}{\pi}\ln\frac{\delta}{1-x}\ : \qquad F(1-x) \sim 1\,, \qquad \tau \longrightarrow 0\,, \end{aligned} \tag{3.17}$$

where $\delta = 2^4$.

The limit $x \to 0$ leads to:

$$\lim_{x\to 0} \mathcal{A}(1_{j_1 I_1}, 2_{j_2 I_2}, 3_{j_3 I_3}, 4_{j_4 I_4}) = g_s \left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\right] \times$$
$$\times \left\{ \frac{\delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3}}{l_s^{-4}\prod_{i=1}^4 R_i} \left[\frac{1}{s} + \sum_{m_i\in\mathbb{Z}-\{0\}} \frac{e^{i2\pi\sum_{i=1}^4 m_i a_i^{I_1 I_3}}\, \delta^{-\sum_{i=1}^4 m_i^2 l_s^2/R_i^2}}{s - \sum_{i=1}^4 \frac{m_i^2}{R_i^2}}\right] + \right.$$
$$\left. + \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3} \left[\sum_{n_i\in\mathbb{Z}} \frac{\delta^{-\sum_{i=1}^4 (n_i + a_i^{I_1 I_3})^2 R_i^2 l_s^{-2}}}{s - \sum_{i=1}^4 (n_i + a_i^{I_1 I_3})^2 R_i^2 l_s^{-4}}\right]\right\}. \tag{3.18}$$

The third line of eq. (3.18) describes the exchange of the lightest (massless when $a_i^{I_1 I_3} = 0$) 33 states as well as their winding excitations with respect to the four ND directions. The numerator is a form factor multiplying the coupling of a 33 state to two 37 states which originates from the interaction of two twisted fields to an untwisted winding mode [4, 17, 18]. Similarly, the second line of eq. (3.18) describes the exchange of the massless 77 states together with their KK excitations. Notice the presence of the $1/R_i$ prefactor which reflects the suppression of the coupling of the D7-brane states compared to those of the D3-branes by the volume of the extra dimensions (bulk).

The exponential form factor of the coupling of KK excitations can be understood from the behavior of D3-branes as solitonic objects with finite thickness inside D7-branes. In fact the interaction of the KK excitations of the D7 gauge fields $A^\mu(x,\vec{y}) = \sum_{\vec{n}} \mathcal{A}^\mu_{\vec{n}} \exp i\frac{n_i y_i}{R_i}$ with the charge density $j_\mu(x)$ associated to massless 37 fermions is described by the effective lagrangian:

$$\int d^4x \sum_{\vec{n}} e^{-\ln\delta \sum_{i=1}^4 \frac{n_i^2 l_s^2}{2R_i^2}}\, j_\mu(x)\, \mathcal{A}^\mu_{\vec{n}}(x)\,, \tag{3.19}$$

which can be written after Fourier transform as

$$\int d^4y \int d^4x \left(\frac{1}{l_s^2 2\pi \ln\delta}\right)^2 e^{-\frac{\vec{y}^2}{2l_s^2\ln\delta}}\, j_\mu(x)\, A^\mu(x,\vec{y})\,. \tag{3.20}$$

It follows that the D3-brane appears from the viewpoint of the D7-brane as a Gaussian distribution of charge $e^{-\frac{\vec{y}^2}{2\sigma^2}} j_\mu(x)$ with a width $\sigma = \sqrt{\ln\delta}\, l_s \sim 1.66\, l_s$. Note the similarity of this result with the numerical estimate of the D-brane width arising from the analysis of tachyon condensation, leading to $\sigma \sim 1.55\, l_s$ [20].

The limit $x \to 1$ of eq. (3.16) can be obtained from (3.18) by exchanging D7-brane indices with the D3-brane ones and s with t:

$$\lim_{x\to 1} \mathcal{A}(1_{j_1 I_1}, 2_{j_2 I_2}, 3_{j_3 I_3}, 4_{j_4 I_4}) = g_s \left[\bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)}\right] \times$$
$$\times \left\{ \delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3} \left[\sum_{n_i\in\mathbb{Z}} \frac{\delta^{-\sum_{i=1}^4 (n_i + a_i^{I_1 I_3})^2 R_i^2 l_s^{-2}}}{t - \sum_{i=1}^4 (n_i + a_i^{I_1 I_3})^2 R_i^2 l_s^{-4}}\right] + \right.$$

$$+\frac{\delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3}}{l_s^{-4}\prod_{i=1}^4 R_i}\left[\frac{1}{t}+\sum_{m_i\in\mathbb{Z}-\{0\}}\frac{e^{i2\pi\sum_{i=1}^4 m_i a_i^{I_1 I_3}}\,\delta^{-\sum_{i=1}^4\frac{m_i^2 l_s^2}{R_i^2}}}{t-\sum_{i=1}^4\frac{m_i^2}{R_i^2}}\right]\Bigg\}\,. \tag{3.21}$$

In the following, we will restrict to the case of maximal gauge symmetry $a_i^I=0$ and we choose all radii to be bigger than the string scale, $R_i > l_s$. The contribution of massless 33 and 77 states as well as the contribution of KK excitations of the latter are computable in the effective field theory. The amplitude receives two types of stringy contributions originating from the exchange of either string winding modes or string oscillators. At low energies, below the string scale, both winding and oscillator states are heavy and can be integrated out to generate effective contact interactions. Their general form is

$$\mathcal{A}^{\rm cont}=\mathcal{A}^{\rm cont}_w+\mathcal{A}^{\rm cont}_{\rm osc}\,, \tag{3.22}$$

where from eqs. (3.18) and (3.21)

$$\begin{aligned}\mathcal{A}^{\rm cont}_w = g_s l_s^2\Big\{&\delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3}\left[\bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)}\right]+\\ &+\delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3}\left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\right]\Big\}\times\\ &\times\sum_{n_i\in\mathbb{Z}-\{0\}}\frac{\delta^{-\sum_{i=1}^4 n_i^2 R_i^2 l_s^{-2}}}{\sum_{i=1}^4 n_i^2 R_i^2 l_s^{-2}}\,.\end{aligned} \tag{3.23}$$

On the other hand,

$$\mathcal{A}^{\rm cont}_{\rm osc}=\lim_{sl_s^2,tl_s^2\to 0}\left\{\mathcal{A}-\mathcal{A}^{\rm QFT}\right\}\qquad\text{with}\qquad \mathcal{A}^{\rm QFT}=(\lim_{x\to 0}+\lim_{x\to 1})\mathcal{A}\,, \tag{3.24}$$

where $\mathcal{A}^{\rm QFT}$ contains also the contribution of 33 string winding modes, in addition to the field theory part from exchanges of massless modes and 77 KK excitations.

3.3 Four ND fermions from three sets of branes

We discuss now the case where the fermions arise from open strings lying on different sets of D7-branes. Without loss of generality, the latter can be chosen to be the second and third sets of table 1, transverse to the 6,7 and 8,9 directions, respectively.

The computation goes as before with the difference that it involves correlations of four twist fields only in the 4th and 5th coordinates while in the remaining four internal directions it involves correlations of two twist fields. The result for the ordered amplitude is

$$\begin{aligned}\mathcal{A}(1_{j_1I_1},2_{j_2I_2},3_{j_3I_3},4_{j_4I_4}) &= -g_s\,l_s^2\int_0^1 dx\,x^{-1-s\,l_s^2}\,(1-x)^{-1-t\,l_s^2}\,\frac{1}{F(x)}\,\times\\ \times\Bigg\{&\delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3}\left[\frac{\bar{u}^{(1)}u^{(2)}\bar{u}^{(4)}u^{(3)}(1-x)}{l_s^{-2}\prod_{i=1}^2 R_i}\sum_{m_i\in\mathbb{Z}}e^{2i\pi\sum_{i=1}^2 m_i a_i^{I_1I_3}}e^{-\pi\tau\sum_i\frac{m_i^2 l_s^2}{R_i^2}}\right.+\end{aligned}$$

$$+ \bar{u}^{(1)}\gamma^\mu u^{(4)}\bar{u}^{(2)}\gamma_\mu u^{(3)}\, x \sum_{n_i\in\mathbb{Z}} e^{-\pi\tau \sum_{i=1}^{2}(n_i+a_i^{I_1 I_3})^2\, R_i^2\, l_s^{-2}} \Bigg] +$$

$$+ \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3} \left[\frac{\bar{u}^{(1)}u^{(4)}\bar{u}^{(2)}u^{(3)}\, x}{l_s^{-2}\prod_{i=1}^{2} R_i} \sum_{m_i\in\mathbb{Z}} e^{2i\pi \sum_{i=1}^{2} m_i a_i^{I_1 I_3}}\, e^{-\pi\tau \sum_i \frac{m_i^2 l_s^2}{R_i^2}} + \right.$$

$$+ \bar{u}^{(1)}\gamma^\mu u^{(2)}\bar{u}^{(4)}\gamma_\mu u^{(3)}\,(1-x) \times$$

$$\left. \times \sum_{n_i\in\mathbb{Z}} e^{-\pi\tau \sum_{i=1}^{2}(n_i+a_i^{I_1 I_3})^2\, R_i^2\, l_s^{-2}} \right]\Bigg\} . \qquad (3.25)$$

Taking the limit of coincident vertices $x \to 0$ or $x \to 1$, one obtains the field theory result which for $a_i^I = 0$ reads:

$$\mathcal{A}^{\rm QFT} = g_s\, \delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3} \times$$

$$\times \left\{ \frac{\left[\bar{u}^{(1)}u^{(2)}\bar{u}^{(4)}u^{(3)}\right]}{l_s^{-2}\prod_{i=1}^{2} R_i} \left[\frac{1}{s} + \sum_{m_i\in\mathbb{Z}-\{0\}} \frac{\delta^{-\sum_{i=1}^{2}\frac{m_i^2 l_s^2}{R_i^2}}}{s - \sum_{i=1}^{2}\frac{m_i^2}{R_i^2}} \right] + \right.$$

$$\left. + \left[\bar{u}^{(1)}\gamma^\mu u^{(4)}\bar{u}^{(2)}\gamma_\mu u^{(3)}\right] \left[\frac{1}{t} + \sum_{n_i\in\mathbb{Z}-\{0\}} \frac{\delta^{-\sum_{i=1}^{2} n_i^2 R_i^2 l_s^{-2}}}{t - \sum_{i=1}^{2} n_i^2 R_i^2 l_s^{-4}} \right] \right\} +$$

$$+ g_s\, \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3} \times$$

$$\times \left\{ \frac{\left[\bar{u}^{(1)}u^{(4)}\bar{u}^{(2)}u^{(3)}\right]}{l_s^{-2}\prod_{i=1}^{2} R_i} \left[\frac{1}{t} + \sum_{m_i\in\mathbb{Z}-\{0\}} \frac{\delta^{-\sum_{i=1}^{2}\frac{m_i^2 l_s^2}{R_i^2}}}{t - \sum_{i=1}^{2}\frac{m_i^2}{R_i^2}} \right] + \right.$$

$$\left. + \left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\right] \left[\frac{1}{s} + \sum_{n_i\in\mathbb{Z}-\{0\}} \frac{\delta^{-\sum_{i=1}^{2} n_i^2 R_i^2 l_s^{-2}}}{s - \sum_{i=1}^{2} n_i^2 R_i^2 l_s^{-4}} \right] \right\} . \qquad (3.26)$$

Note that only the KK excitations and the winding modes associated with the two internal dimensions longitudinal to both D7-branes are exchanged among the four fermions. Moreover, while the exchanged 33 states are vector bosons, the exchanged 77' states are spin 0 particles, thus the absence of γ_μ-matrices.

3.4 Two ND and two DD fermions

For completeness we also present the last possibility where only two of the fermions arise from ND open strings stretched between the same set of branes. The computation is straightforward [21] and gives a result similar to the case of four DD fermions:

$$\mathcal{A}(1_{j_1 I_1}, 2_{j_2 I_2}, 3, 4) = -2 g_s\, l_s^2\, \{\lambda^3, \lambda^4\}_{I_1 \bar{I}_2}\delta_{j_1,\bar{j}_2} \int_0^1 dx\, x^{-1-s\, l_s^2}\,(1-x)^{-1-t\, l_s^2} \times$$

$$\times \left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}(1-x) + \bar{u}^{(1)}u^{(4)}\bar{u}^{(2)}u^{(3)} x\right] ,$$

where the absence of γ-matrices in the t channel is due to the fact that it corresponds to the exchange of 37 open strings which do not contain spin-one modes. The case of two ND and two DD fermions is thus very similar to the one of four DD strings given in eq. (3.8); the effective action does not contain dimension-six contact interactions but the leading correction comes from dimension-eight operators.

4. Effective four-fermion contact interactions

In this section we would like to compute the contact interaction $\mathcal{A}^{\text{contact}}$ between four fermions induced by massive string states: oscillators and winding modes. We will first study the case of four ND fermions from two sets of branes where the relevant amplitudes have been defined in eqs. (3.22) to (3.24). The other cases will be discussed in section 4.3.

4.1 Two sets of branes: the infinite transverse volume case

We first consider the simplest case obtained by taking the size of the directions appearing in the result of the correlation functions (3.14) of twist fields to be very large: $R_i/l_s \to \infty$. In this limit the contribution of winding modes $\mathcal{A}_w^{\text{cont}}$ given by (3.23) vanishes and the contact interaction is due only to massive string oscillation modes $\mathcal{A}^{\text{cont}} \simeq \mathcal{A}_{\text{osc}}^{\text{cont}}$ given in eq. (3.24). The field theory part $\mathcal{A}^{\text{QFT}}$ is due to the exchange of massless 33 modes:

$$\begin{aligned}\mathcal{A}^{33} = {} & \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,J_4}\delta_{I_2,\bar{I}_3}\left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\right]\frac{g_s}{s} + \\ & + \delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3}\left[\bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)}\right]\frac{g_s}{t}\,,\end{aligned} \tag{4.1}$$

as well as to the exchange of massless 77 states and their KK excitations:

$$\begin{aligned}\mathcal{A}^{77} = {} & \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3}\frac{\left[\bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)}\right]}{\prod_i (R_i/l_s)}\sum_{m_i}\frac{g_s\,\delta^{-\sum_i \frac{m_i^2 l_s^2}{R_i^2}}}{t-\sum_i \frac{m_i^2}{R_i^2}} + \\ & + \delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3}\frac{\left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\right]}{\prod_i (R_i/l_s)}\sum_{m_i}\frac{g_s\,\delta^{-\sum_i \frac{m_i^2 l_s^2}{R_i^2}}}{s-\sum_i \frac{m_i^2}{R_i^2}}\,,\end{aligned} \tag{4.2}$$

where $i = 1, \ldots, 4$.

The contact interaction is then given by:

$$\mathcal{A}^{\text{contact}}|_{R_i/l_s\to\infty} = \mathcal{A}|_{R_i/l_s\to\infty} - \mathcal{A}^{33} - \mathcal{A}^{77}\,. \tag{4.3}$$

Notice that each term in the r.h.s. of eq. (4.3) has two contributions proportional to $\delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3}$ and to $\delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3}$. These contributions are related by the exchange of the states 2 and 4, which takes s to t. To verify this property in the integral representation of the amplitude (3.16) one should change x to $1-x$ and perform a Poisson ressumation. We can thus restrict our analysis to the terms proportional to $\delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3}$.

The ordered amplitude $\mathcal{A}$ in (3.16) contains the factor $\sum_{n_i} e^{-\pi\tau\sum_i n_i^2 R_i^2 l_s^{-2}}$ which vanishes exponentially in the limit $R_i/l_s \to \infty$ unless $n_i = 0$ or $\tau \to 0$. The limit $\tau \to 0$ corresponds to $x \to 1$, as it can be seen from (3.17), and produces the field theory result $\mathcal{A}^{77}$ of eq. (4.2) which we must subtract according to eq. (4.3). We are left over with the contribution of the zero mode $n_i = 0$, given by the integral (3.16) after substracting the integration region $x \to 1$:

$$
\begin{aligned}
-g_s l_s^2 \int_0^1 dx \Bigg\{ & x^{-1-sl_s^2}(1-x)^{-1-tl_s^2} \times \\
& \times \frac{\left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}(1-x) + \bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)} x\right]}{[F(x)]^2} - \\
& - (1-x)^{-1-tl_s^2}\left[\bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)}\right]\left[\frac{\pi}{\ln\frac{\delta}{1-x}}\right]^2 \Bigg\} .
\end{aligned} \tag{4.4}
$$

This contains a simple pole in s due to the exchange of massless 33 strings given in eq. (4.1). Following eq. (4.3) this pole should also be substracted by adding to (4.4) the term

$$
g_s\, l_s^2 \int_0^1 dx\, x^{-1-s\, l_s^2} \left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\right] . \tag{4.5}
$$

Adding (4.4) and (4.5) and taking the low energy limit $|s\, l_s^2| \to 0$ and $|t\, l_s^2| \to 0$ we obtain the dimension-six effective operator:

$$
\begin{aligned}
& -g_s l_s^2 \left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\right] \int_0^1 \frac{dx}{x}\left(\frac{1}{[F(x)]^2} - 1\right) - \\
& -g_s l_s^2 \left[\bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)}\right] \int_0^1 \frac{dx}{1-x}\left(\frac{1}{[F(x)]^2} - \left[\frac{\pi}{\ln\frac{\delta}{1-x}}\right]^2\right) .
\end{aligned} \tag{4.6}
$$

To give a numerical evaluation of the above four-fermion interaction, it is convenient to make a change of variables from x to τ using eq. (3.15). The integration domain is then divided into two regions: $\tau \in [0,1]$ and $\tau \in [1,\infty]$. To make the convergence of the integrals manifest we perform a transformation $\tau \to 1/\tau$ in the region $\tau \in [0,1]$ so that the integration is now only from 1 to ∞. Adding the two contributions proportional to $\delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3}$ and $\delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3}$, we obtain for the four-fermion contact term the following result:

$$
\begin{aligned}
\mathcal{A}^{\text{contact}} &= -\delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3}\, g_s\, l_s^2 \Bigg\{ \left[\bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)}\right] \times \\
&\times \pi \int_1^\infty d\tau \left(\frac{\theta_2^4(i\tau)}{\theta_3^4(i\tau)} + \frac{1}{\tau^2}\frac{\theta_4^4(i\tau)}{\theta_3^4(i\tau)} - \theta_2^4(i\tau)\left[\frac{\pi}{4\ln(2\frac{\theta_3}{\theta_4})}\right]^2 - \theta_4^4(i\tau)\left[\frac{\pi}{4\ln(2\frac{\theta_3}{\theta_2})}\right]^2 \right) + \\
&+ \left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\right] \pi \int_1^\infty d\tau \left(\frac{\theta_4^4(i\tau)}{\theta_3^4(i\tau)} - \theta_3^4(i\tau) + \frac{1}{\tau^2}\frac{\theta_2^4(i\tau)}{\theta_3^4(i\tau)} \right) \Bigg\} - \\
&- \delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3} g_s l_s^2 \Bigg\{ \left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\right] \times \\
&\times \pi \int_1^\infty d\tau \left(\frac{\theta_2^4(i\tau)}{\theta_3^4(i\tau)} + \frac{1}{\tau^2}\frac{\theta_4^4(i\tau)}{\theta_3^4(i\tau)} - \theta_2^4(i\tau)\left[\frac{\pi}{4\ln(2\frac{\theta_3}{\theta_4})}\right]^2 - \theta_4^4(i\tau)\left[\frac{\pi}{4\ln(2\frac{\theta_3}{\theta_2})}\right]^2 \right) + \\
&+ \left[\bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)}\right] \pi \int_1^\infty d\tau \left(\frac{\theta_4^4(i\tau)}{\theta_3^4(i\tau)} - \theta_3^4(i\tau) + \frac{1}{\tau^2}\frac{\theta_2^4(i\tau)}{\theta_3^4(i\tau)} \right) \Bigg\} \qquad (4.7)
\end{aligned}
$$

which can be approximated by numerical integration as:

$$
\begin{aligned}
\mathcal{A}^{\text{contact}} \simeq\ & g_s l_s^2 \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3}\Big\{0.20\bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)} + \\
&\qquad + 0.59\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\Big\} + \\
&+ g_s l_s^2 \delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3}\Big\{0.59\bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)} + \\
&\qquad + 0.20\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\Big\}. \qquad (4.8)
\end{aligned}
$$

In eq. (4.8), the leading contribution (with coefficient 0.59) arises from exchange of massive oscillators of 33 states and will be used to extract experimental bounds on the string scale in section 6.

Note that the coefficients of the above contact operators are positive i.e. opposite to those due to the exchange of massive KK excitations of gauge bosons. The non-vanishing effective operators for fermions of the same type (all the D3- and D7-brane indices equal $\delta_{I_a,\bar{I}_b} = \delta_{\bar{j}_a,j_b} = 1$) with fixed helicities are:

$$
\begin{aligned}
\mathcal{A}(1_L^-, 2_R^+, 3_L^-, 4_R^+) &\simeq 3.16\, g_s\, u l_s^2\,, \\
\mathcal{A}(1_L^-, 2_R^+, 3_R^-, 4_L^+) &\simeq -1.58\, g_s\, t l_s^2\,, \\
\mathcal{A}(1_L^-, 2_L^+, 3_L^-, 4_L^+) &\simeq -1.58\, g_s\, s l_s^2\,, \qquad (4.9)
\end{aligned}
$$

while the other non-vanishing operators can be obtained by parity reflection.

4.2 Two sets of branes: generalization to arbitrary internal radii

We consider now the generic case where the sizes of d directions associated with the KK and winding modes exchanged between the four fermions are kept finite while the remaining are taken infinitely large. Following the definition of $\mathcal{A}^{\rm contact}$ in (3.22), we substract the $\mathcal{A}^{33}$ and $\mathcal{A}^{77}$ contributions from the total ordered amplitude; this gives for the term proportional to $\delta_{I_1,\bar I_2}\delta_{I_3,\bar I_4}\delta_{\bar j_1,j_4}\delta_{j_2,\bar j_3}$:

$$\begin{aligned}
\mathcal{A}^{\rm contact} &= -g_s\, l_s^2 \left[\bar u^{(1)}\gamma_\mu u^{(2)}\bar u^{(4)}\gamma^\mu u^{(3)}\right] \int_0^1 \frac{dx}{x}\left(\frac{1}{[F(x)]^2}\sum_{n_i} e^{-\pi\tau\sum_{i=1}^d \frac{n_i^2 R^2}{l_s^2}} - 1\right) - \\
&\quad - g_s\, l_s^2 \left[\bar u^{(1)}\gamma_\mu u^{(4)}\bar u^{(2)}\gamma^\mu u^{(3)}\right] \int_0^1 \frac{dx}{1-x}\times \\
&\quad \times\left(\frac{1}{[F(x)]^2}\sum_{n_i} e^{-\pi\tau\sum_{i=1}^d \frac{n_i^2 R_i^2}{l_s^2}} - \frac{l_s^d}{\prod_{i=1}^d R_i}\left[\frac{\pi}{\ln\left(\frac{\delta}{1-x}\right)}\right]^{\frac{4-d}{2}}\times\right. \\
&\quad \left.\times\sum_{n_i}\left[\frac{(1-x)}{\delta}\right]^{\sum_{i=1}^d \frac{n_i^2 l_s^2}{R_i^2}}\right), \qquad (4.10)
\end{aligned}$$

where in the last term inside the brackets, corresponding to the $x \to 1$ asymptotic region, we performed a Poisson resummation. The above contact term contains contributions of both string oscillators and winding states along the finite size directions, in contrast to the large radius limit for all directions that we considered in section 4.1. The contribution of the winding modes reads:

$$\mathcal{A}_w^{\rm cont} = -g_s\, l_s^2\, \left[\bar u^{(1)}\gamma_\mu u^{(2)}\bar u^{(4)}\gamma^\mu u^{(3)}\right] \sum_{n_i\in\mathbb{Z}-\{0\}} \frac{\delta^{-\sum_{i=1}^d n_i^2\, R_i^2\, l_s^{-2}}}{\sum_{i=1}^d n_i^2\, R_i^2\, l_s^{-2}}\,. \qquad (4.11)$$

To study the dependance of the contact term (4.10) on the size of the compactification space we consider the simplest case of one finite size dimension and three infinite ones ($d = 1$). As in the case of infinite radius, for the numerical evaluations it is useful to express the corresponding integral as a function of τ. The resulting expression is:

$$\begin{aligned}
\mathcal{A}^{\rm contact}(R) &= -g_s\, l_s^2\, \delta_{I_1,\bar I_2}\delta_{J_3,\bar J_4}\delta_{\bar j_1,j_4}\delta_{j_2,\bar j_3}\times \\
&\quad \times\Bigg[\left[\bar u^{(1)}\gamma_\mu u^{(4)}\bar u^{(2)}\gamma^\mu u^{(3)}\right]\pi\times \\
&\quad \times\int_1^\infty d\tau\left(\frac{\theta_2^4}{\theta_3^4}(i\tau)\theta_3\left(i\frac{R^2\tau}{l_s^2}\right) + \frac{l_s}{R\tau^{3/2}}\frac{\theta_4^4}{\theta_3^4}(i\tau)\theta_3\left(i\frac{l_s^2\tau}{R^2}\right) - \right. \\
&\quad \left. -\ \theta_2^4(i\tau)\frac{l_s}{R}\left[\frac{\pi}{4\ln\left(2\frac{\theta_3}{\theta_4}(i\tau)\right)}\right]^{3/2}\sum_{n\in\mathbb{Z}}\left[\frac{\theta_4}{2\theta_3}(i\tau)\right]^{4n^2 l_s^2/R^2} - \right.
\end{aligned}$$

$$
\begin{aligned}
&- \theta_4^4(i\tau)\frac{l_s}{R}\left[\frac{\pi}{4\ln\left(2\frac{\theta_3}{\theta_2}(i\tau)\right)}\right]^{3/2}\sum_{n\in\mathbb{Z}}\left[\frac{\theta_2}{2\theta_3}(i\tau)\right]^{4n^2l_s^2/R^2}\Bigg) + \\
&+ \left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\right]\pi\int_1^\infty d\tau\,\left(\frac{\theta_4^4}{\theta_3^4}(i\tau)\theta_3\left(i\frac{R^2\tau}{l_s^2}\right) - \theta_3^4(i\tau) + \right. \\
&\qquad \left.+ \frac{l_s}{R\tau^{3/2}}\frac{\theta_2^4}{\theta_3^4}(i\tau)\theta_3\left(i\frac{l_s^2\tau}{R^2}\right)\right)\Bigg] - \\
&- g_s\, l_s^2\delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,J_4}\delta_{I_2,\bar{J}_3}\times \\
&\times\Bigg\{[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}]\pi\times \\
&\times\int_1^\infty d\tau\,\left(\frac{\theta_2^4}{\theta_3^4}(i\tau)\theta_3\left(i\frac{R^2\tau}{l_s^2}\right) + \frac{l_s}{R\tau^{3/2}}\frac{\theta_4^4}{\theta_3^4}(i\tau)\theta_3\left(i\frac{l_s^2\tau}{R^2}\right) - \right. \qquad (4.12) \\
&- \theta_2^4(i\tau)\frac{l_s}{R}\left[\frac{\pi}{4\ln\left(2\frac{\theta_3}{\theta_4}(i\tau)\right)}\right]^{3/2}\sum_{n\in\mathbb{Z}}\left[\frac{\theta_4}{2\theta_3}(i\tau)\right]^{\frac{4n^2l_s^2}{R^2}} - \\
&\left. - \theta_4^4(i\tau)\frac{l_s}{R}\left[\frac{\pi}{4\ln\left(2\frac{\theta_3}{\theta_2}(i\tau)\right)}\right]^{3/2}\sum_{n\in\mathbb{Z}}\left[\frac{\theta_2}{2\theta_3}(i\tau)\right]^{\frac{4n^2l_s^2}{R^2}}\right) + \\
&+ [\bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)}]\pi\int_1^\infty d\tau\,\left(\frac{\theta_4^4}{\theta_3^4}(i\tau)\theta_3\left(i\frac{R^2\tau}{l_s^2}\right) - \theta_3^4(i\tau) + \right. \\
&\qquad \left. + \frac{l_s}{R\tau^{3/2}}\frac{\theta_2^4}{\theta_3^4}(i\tau)\theta_3\left(i\frac{l_s^2\tau}{R^2}\right)\right)\Bigg\}.
\end{aligned}
$$

The behavior as a function of R/l_s of the contact term due to massive 33 string oscillator states (given by the first line of eq. (4.10) or equivalently by the integral in the 5th and 6th lines of eq. (4.12)) is shown in figure 1 (solid line). In the limit $R/l_s \to \infty$, it reaches rapidly its asymptotic value 0.59 of eq. (4.8), while when $R/l_s \to 0$, it diverges as $-\pi^2 l_s^2/3R^2$, obtained from the sum (4.11) with $d = 1$. This limit corresponds, upon T-duality $R \to l_s^2/R$, to the decompactification of a longitudinal dimension along the world-volume of a D4-brane.

4.3 Three sets of branes

In this case, there are two different sets of 7-branes and the corresponding amplitude is given in eq. (3.25). The difference with the previous case is that there are two (instead of four) internal directions that appear in the amplitude and that the 77' channel leads only to scalar exchanges. For d arbitrary internal radii ($d \leq 2$), the contribution proportional to $\delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3}$, previously given in eq. (4.10),

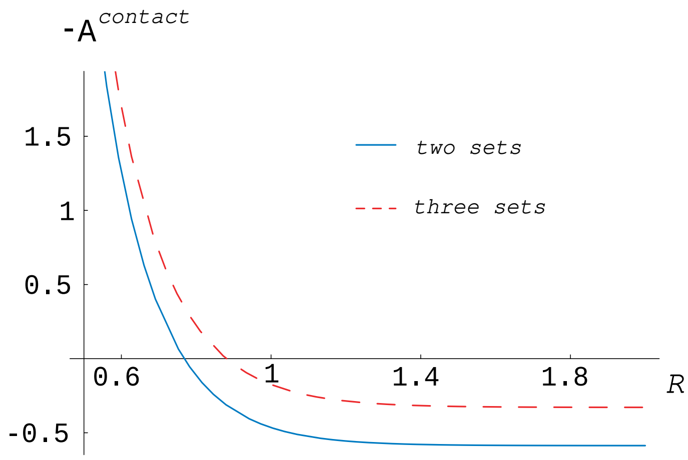

Figure 1: Radius dependence of the four-fermion contact terms for two (solid line) and three (dashed line) sets of branes.

becomes:

$$\begin{aligned}\mathcal{A}^{\rm contact} &= -g_s\, l_s^2\, [\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}] \int_0^1 \frac{dx}{x}\left(\frac{1}{F(x)}\sum_{n_i} e^{-\pi\tau\sum_{i=1}^d \frac{n_i^2 R^2}{l_s^2}} - 1\right) - \\ &\quad - g_s\, l_s^2\, [\bar{u}^{(1)} u^{(4)}\bar{u}^{(2)} u^{(3)}] \times \\ &\quad \times \int_0^1 \frac{dx}{1-x}\left(\frac{1}{F(x)}\sum_{n_i} e^{-\pi\tau\sum_{i=1}^d \frac{n_i^2 R_i^2}{l_s^2}} - \right. \\ &\qquad \left. - \frac{l_s^d}{\prod_{i=1}^d R_i}\left[\frac{\pi}{\ln\left(\frac{\delta}{1-x}\right)}\right]^{\frac{2-d}{2}} \sum_{n_i}\left[\frac{(1-x)}{\delta}\right]^{\sum_{i=1}^d \frac{n_i^2 l_s^2}{R_i^2}}\right) . \end{aligned} \tag{4.13}$$

In figure 1, we plot (dashed line) the radius dependence of the contact term due to the exchange of 33 string states (first line of eq. (4.13)) in the case where only one of the two radii appearing in the amplitude is kept finite. In the infinite radius limit, we obtain:

$$\begin{aligned}\mathcal{A}^{\rm contact} &= -g_s\, l_s^2\, \left[\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\right] \int_0^1 \frac{dx}{x}\left(\frac{1}{F(x)} - 1\right) - \\ &\quad - g_s\, l_s^2\, \left[\bar{u}^{(1)} u^{(4)}\bar{u}^{(2)} u^{(3)}\right] \int_0^1 \frac{dx}{1-x}\left(\frac{1}{F(x)} - \frac{\pi}{\ln\frac{\delta}{1-x}}\right) . \end{aligned} \tag{4.14}$$

Performing a change of variables, the above integrands can be expressed as a function of τ and the integrals can be evaluated numerically with the result:

$$\begin{aligned}\mathcal{A}^{\rm contact} &\simeq g_s l_s^2 \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3}\left\{0.12\bar{u}^{(1)}u^{(4)}\bar{u}^{(2)}u^{(3)} + 0.33\bar{u}^{(1)}\gamma_\mu u^{(2)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\right\} + \\ &\quad + g_s\, l_s^2\, \delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3}\left\{\, 0.33\,\bar{u}^{(1)}\gamma_\mu u^{(4)}\bar{u}^{(2)}\gamma^\mu u^{(3)} + \right. \\ &\qquad \left. + \, 0.12\,\bar{u}^{(1)}u^{(2)}\bar{u}^{(4)}u^{(3)}\right\} , \end{aligned} \tag{4.15}$$

where the leading contributions (coefficient 0.33) are due to the exchange of massive oscillators of 33 open strings. On the other hand, in the limit $R/l_s \to 0$, one finds the divergent behavior $-\pi^2 l_s^2/3R^2$, as in the case of two sets of branes.

5. Summary of the effective field theory results

In this section we would like to summarize the results for the effective contact interactions obtained in the previous sections.

In the case where the four-fermion scattering involves two or four states arising from DD open strings with both ends on the same set of branes, we have found that the final amplitude can be written as:

$$\mathcal{A}(s,t) = \mathcal{A}_{\rm point}(s,t) \cdot \mathcal{F}(s,t)\,, \tag{5.1}$$

where $\mathcal{A}_{\rm point}$ is the result of the (two derivative) low energy effective action, while

$$\mathcal{F}(s,t) = \frac{\Gamma(1-l_s^2 s)\Gamma(1-l_s^2 t)}{\Gamma(1-l_s^2 s - l_s^2 t)} = 1 - \frac{\pi^2}{6}\frac{st}{M_S^4} + \cdots \tag{5.2}$$

represents the string form factor correction. This implies that the leading contact term between the four fermions is of dimension-eight. The case of four DD strings can be understood from the fact that the tree-level interactions are obtained by a truncation of the $N=4$ supersymmetric theory arising on D3-branes, upon compactification of the ten-dimensional theory. Supersymmetry relates the DD fermions to gauge fields and there is no dimension-six operator because of the absence of F^3 terms with F the gauge field strength.

The cases involving four ND strings, describing states localized on brane intersections, have a richer structure. We have found that the effective field theory contains dimension-six four-fermion contact terms which get contributions from three different sources:

- *Exchange of KK excitations:* consider for simplicity the case of two sets of branes, one corresponding to D3-branes and the other to D7-branes. Then, the exchange of KK excitations of 77 (bulk) states leads to dimension-six operators of the form:

$$\begin{aligned} &- \left[\bar\psi^{(1)}\gamma_{(6)M}\psi^{(2)}\bar\psi^{(4)}\gamma^{(6)M}\psi^{(3)}\right] \frac{g_s}{l_s^{-4}\prod_{i=1}^4 R_i} \times \\ &\times \sum_{m_i\in\mathbb{Z}-\{0\}} \frac{e^{i2\pi\sum_{i=1}^4 m_i a_i^{I_1 I_3}}\,\delta^{-\sum_{i=1}^4 \frac{m_i^2 l_s^2}{R_i^2}}}{\sum_{i=1}^4 \frac{m_i^2}{R_i^2}}\,, \end{aligned} \tag{5.3}$$

  where $\delta = 2^4$ and $\gamma_{(6)}^M$ stand for the γ-matrices in the six-dimensional spacetime defined by the directions that are parallel or transverse simultaneously to the

D3 and D7-branes. Reduction to four dimensions leads to both gauge and Yukawa couplings. This expression is identical to the one obtained in heterotic orbifolds with large extra dimensions [4].

The main features of the above result are: (i) the suppresion of the gauge coupling by the internal volume felt by the bulk states propagating on the D7-branes; (ii) the exponential suppression of the coupling of KK excitations (with masses $m_{\rm KK}$) to the massless localized fermions by the factor $\delta^{-m_{\rm KK}^2/2M_s^2}$. This factor can be interpreted as a finite width for a D3-brane, $\sigma = \sqrt{\ln\delta}\, l_s \sim 1.66\, l_s$ [see eq. (3.20)]; (iii) the appearance of phases $e^{2i\pi\sum_{i=1}^4 m_i a_i^{I_1 I_3}}$ when the localized fermions are separated by the distance $2\pi a_i^{I_1 I_3} R_i$ along the i-th direction [4].

- *Exchange of winding modes of the 33 states:* these lead to an operator of the form:

$$ - \left[\bar\psi^{(1)}\gamma_M^{(6)}\psi^{(4)}\bar\psi^{(2)}\gamma^{(6)M}\psi^{(3)}\right] g_s \left[\sum_{n_i\in\mathbb{Z}} \frac{\delta^{-\sum_{i=1}^4 (n_i + a_i^{I_1I_3})^2 R_i^2 l_s^{-2}}}{\sum_{i=1}^4 (n_i + a_i^{I_1I_3})^2 R_i^2 l_s^{-4}}\right], \tag{5.4}$$

where in the case of vanishing Wilson lines $a_i^{I_1I_3} = 0$, the pole due to the exchange of massless states should be substracted to obtain the contact term. The operator of (5.4) is exponentially suppressed in the large radius limit, due to the exchange of massive open strings streched between the D3-branes.

- *Exchange of massive oscillators:* their contribution can be simplified by taking the large (internal) volume limit in the absence of Wilson lines. It is also useful to separate the contributions associated with the exchanges of 33 and 77 string states. In the case where all four fermions arise from open strings ending on the same sets of D3- and D7-branes, we find:

$$\begin{aligned} \mathcal{A}_{33}^{\rm contact} &= -g_s\, l_s^2 \int_0^1 \frac{dx}{x}\left(\frac{1}{[F(x)]^2} - 1\right) \times \\ &\quad \times \Big\{ \delta_{j_1,\bar j_2}\delta_{j_3,\bar j_4}\delta_{\bar I_1,J_4}\delta_{I_2,\bar I_3}\left[\bar\psi^{(1)}\gamma_M^{(6)}\psi^{(2)}\bar\psi^{(4)}\gamma^{(6)M}\psi^{(3)}\right] - \\ &\qquad - \delta_{I_1,\bar I_2}\delta_{I_3,\bar I_4}\delta_{\bar j_1,j_4}\delta_{j_2,\bar j_3}\left[\bar\psi^{(1)}\gamma_M^{(6)}\psi^{(4)}\bar\psi^{(2)}\gamma^{(6)M}\psi^{(3)}\right]\Big\}, \end{aligned} \tag{5.5}$$

where I_a and j_a are indices labelling, respectively, the D3- and D7-branes. A numerical estimate gives:

$$\begin{aligned} \mathcal{A}_{33}^{\rm contact} &\simeq 0.59\, g_s\, l_s^2 \Big\{ \delta_{j_1,\bar j_2}\delta_{j_3,\bar j_4}\delta_{\bar I_1,I_4}\delta_{I_2,\bar I_3}\,\bar\psi^{(1)}\gamma_M^{(6)}\psi^{(2)}\bar\psi^{(4)}\gamma^{(6)M}\psi^{(3)} + \\ &\qquad + \delta_{I_1,\bar I_2}\delta_{I_3,\bar I_4}\delta_{\bar j_1,j_4}\delta_{j_2,\bar j_3}\,\bar\psi^{(1)}\gamma_M^{(6)}\psi^{(4)}\bar\psi^{(2)}\gamma^{(6)M}\psi^{(3)}\Big\}. \end{aligned} \tag{5.6}$$

In the channel corresponding to exchange of 77 states we obtain:

$$\mathcal{A}_{77}^{\rm contact} = -g_s\, l_s^2 \int_0^1 \frac{dx}{1-x}\left(\frac{1}{[F(x)]^2} - \left[\frac{\pi}{\ln\frac{\delta}{1-x}}\right]^2\right) \times$$

$$
\times \left\{ \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3} \left[\bar{\psi}^{(1)}\gamma_M^{(6)}\psi^{(4)}\bar{\psi}^{(2)}\gamma^{(6)M}\psi^{(3)} \right] + \right.
$$

$$
\left. + \delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3} \left[\bar{\psi}^{(1)}\gamma_M^{(6)}\psi^{(2)}\bar{\psi}^{(4)}\gamma^{(6)M}\psi^{(3)} \right] \right\} \qquad (5.7)
$$

which can be approximated by:

$$
\mathcal{A}_{77}^{\rm contact} \simeq 0.20 g_s\, l_s^2 \left\{ \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3} \left[\bar{\psi}^{(1)}\gamma_M^{(6)}\psi^{(4)}\bar{\psi}^{(2)}\gamma^{(6)M}\psi^{(3)} \right] + \right.
$$

$$
\left. + \delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3} \left[\bar{\psi}^{(1)}\gamma_M^{(6)}\psi^{(2)}\bar{u}^{(4)}\gamma^{(6)M}\psi^{(3)} \right] \right\}. \qquad (5.8)
$$

Another possibility arises when there are two different sets of D7-branes, say D7 and D7', so that one pair of fermions corresponds to 37-strings stretched between the D3- and D7-branes while the other pair corresponds to 37' strings. In this case, the contact interactions due to 33-exchanges are:

$$
\mathcal{A}_{33}^{\rm contact} = -g_s\, l_s^2 \int_0^1 \frac{dx}{x} \left(\frac{1}{F(x)} - 1 \right) \times
$$

$$
\times \left\{ \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,J_4}\delta_{I_2,\bar{I}_3} \left[\bar{\psi}^{(1)}\gamma_\mu\psi^{(2)}\bar{\psi}^{(4)}\gamma^\mu\psi^{(3)} \right] - \right.
$$

$$
\left. - \; \delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3} \left[\bar{\psi}^{(1)}\gamma_\mu\psi^{(4)}\bar{\psi}^{(2)}\gamma^\mu\psi^{(3)} \right] \right\} \qquad (5.9)
$$

which can be numerically evaluated:

$$
\mathcal{A}_{33}^{\rm contact} \simeq 0.33 g_s\, l_s^2 \left\{ \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3}\, \bar{\psi}^{(1)}\gamma_\mu\psi^{(2)}\bar{\psi}^{(4)}\gamma^\mu\psi^{(3)} + \right.
$$

$$
\left. + \; \delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3}\, \bar{\psi}^{(1)}\gamma^\mu\psi^{(4)}\bar{\psi}^{(2)}\gamma_\mu\psi^{(3)} \right\}, \qquad (5.10)
$$

Similarly, the exchange of $77'$ states gives:

$$
\mathcal{A}_{77'}^{\rm contact} = -g_s\, l_s^2 \int_0^1 \frac{dx}{1-x} \left(\frac{1}{F(x)} - \frac{\pi}{\ln \frac{\delta}{1-x}} \right) \times
$$

$$
\times \left\{ \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3} \left[\bar{\psi}^{(1)}\psi^{(4)}\bar{\psi}^{(2)}\psi^{(3)} \right] + \right.
$$

$$
\left. + \; \delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3} \left[\bar{\psi}^{(1)}\psi^{(2)}\bar{\psi}^{(4)}\psi^{(3)} \right] \right\} \qquad (5.11)
$$

which can be approximated by:

$$
\mathcal{A}_{77'}^{\rm contact} \simeq 0.12\, g_s\, l_s^2 \left\{ \delta_{j_1,\bar{j}_2}\delta_{j_3,\bar{j}_4}\delta_{\bar{I}_1,I_4}\delta_{I_2,\bar{I}_3} \left[\bar{\psi}^{(1)}\psi^{(4)}\bar{\psi}^{(2)}\psi^{(3)} \right] \right.
$$

$$
\left. + \; \delta_{I_1,\bar{I}_2}\delta_{I_3,\bar{I}_4}\delta_{\bar{j}_1,j_4}\delta_{j_2,\bar{j}_3} \left[\bar{\psi}^{(1)}\psi^{(2)}\bar{\psi}^{(4)}\psi^{(3)} \right] \right\}. \qquad (5.12)
$$

It is important to notice that the contractions of Chan-Paton indices in the above formulae are associated with exchanges of $\mathrm{U}(N)$ states and not of $\mathrm{SU}(N)$. In a realistic model some of the $\mathrm{U}(1)$ factors are anomalous. The anomalies are canceled by a generalized Green-Schwarz mechanism which provides the corresponding U(1) gauge bosons with masses of the order of the string scale [22]. While these U(1)'s do not appear in the low energy degrees of freedom, they contribute in general to the effective four-fermion operators.

In the generic case with finite radii, one can substract the effects of exchanges of KK excitations that are easily computed in the field theory limit, to find an effective contact interaction which contains the combined effects of massive string oscillators and winding modes. The result is ploted in figure 1 as a function of one compactification radius. We observe that the size of the operator goes quickly to its asymptotic value of infinite volume, already for a radius of order $R \sim 1.4\, l_s$. On the other hand, in the limit $R/l_s \to 0$, one recovers the asymptotic value due to the sum of the KK exchanges of masses mR/l_s^2. This is given in eq. (5.3) by replacing $R \to l_s^2/R$ (T-duality).

6. Experimental bounds from contact terms

Above, we have shown that the exchange of massive open string states leads to dimension-six effective operators among four fermions localized on brane intersections. These operators are generically parametrized as [23]:

$$\Lambda_{\rm eff} = \frac{4\pi}{(1+\varepsilon)\Lambda^2} \sum_{a,b=L,R} \eta_{ab} \bar{\psi}_a \gamma^\mu \psi_a \bar{\psi}'_b \gamma_\mu \psi'_b \tag{6.1}$$

with $\varepsilon = 1$ (0) for $\psi = \psi'$ ($\psi \neq \psi'$), where ψ_a and ψ'_b are left (L) or right (R) handed spinors. Λ is the scale of contact interactions and η_{ab} parametrize the relative strengths of various helicity combinations. In D-brane models there are in general three types of contact terms due to the exchanges of either massive KK excitations, or massive winding modes or string oscillator states.

In order to have a clear identification of longitudinal and transverse directions with respect to the D-branes, we choose all internal radii to be larger than the string length. For concretness, we first consider the simplest scenario where the standard model degrees of freedom arise from D3-branes with matter fields coming from 37 open strings having only one end on the D3-branes. Then, the bulk 77 states have new (exotic) quantum numbers and interact with the 37 matter fields through volume suppressed couplings. Since these interactions are model dependent we will not consider them in our subsequent analysis. The remaining contact terms originate from the exchange of either massive winding modes of the 33 states or string oscillator modes. The winding contributions are exponentially suppressed in the large (internal) volume limit, and we are left with the contribution of the massive string states. The result is given in eqs. (5.6) and (5.8) for the case of two sets of branes and in eqs. (5.10) and (5.12) for the case of three sets.

For $\psi \neq \psi'$ one can choose the quantum numbers (i.e. Chan-Paton indices) of the 37 strings such that the operator (6.1) receives contributions only from the exchange of the massive 33 open string states. Indeed, by choosing the indices so that only the first contraction of eqs. (5.6) and (5.8) survives, and identifying $u^{(1)}$, $u^{(2)}$ with ψ and $u^{(3)}$, $u^{(4)}$ with ψ', one obtains:

$$\begin{aligned}\Lambda_{\rm eff}^{\rm contact} \simeq \frac{g_s}{M_s^2}\Bigg\{&0.59 \sum_{a,b=L,R} \eta_{ab}\bar{\psi}_a\gamma^\mu\psi_a\bar{\psi}'_b\gamma_\mu\psi'_b + \\ &+ 0.20 \sum_{a,b=L,R} \eta_{ab}\bar{\psi}_a\gamma^\mu\psi'_a\bar{\psi}_b\gamma_\mu\psi'_b\Bigg\}\,,\end{aligned} \tag{6.2}$$

where the two terms correspond to the exchanges of 33 and 77 open string states, respectively. Thus, the first operator which coincides with (6.1) receives contributions only from the exchange of massive 33 open string oscillators. Exchanges of 77 states contribute to the second operator which induces new (flavor changing) interactions that are model dependent and must be suppressed in a realistic model. In fact, it is easy to see that there is a choice of spinor helicities $(\psi_L, \psi_L, \psi'_R, \psi'_R)$ which eliminates the second operator completely. The same analysis holds for the case where the external fermions arise from three different sets of branes with eq. (6.2) replaced by the first line of eqs. (5.10) and (5.12).

It follows that one can identify the parameters in eq. (6.1) as:

$$\begin{aligned}\eta_{LL} = \eta_{RR} = \eta_{LR} = \eta_{RL} = 1\,,\\ \Lambda \simeq \sqrt{\frac{4\pi}{0.59 g_s}} M_s\end{aligned} \tag{6.3}$$

in the case of two sets of banes, while for 3 sets 0.59 is replaced by 0.33. The signs and relative ratios of the different terms in (6.1) correspond to what is usually refered to as Λ^+_{VV}. The present bounds from LEP [25] are of the order of $\Lambda^+_{VV} \gtrsim 16\,\text{TeV}$ which for $g_s = g^2_{\rm YM} \sim 1/2$, with $g_{\rm YM}$ the gauge coupling, leads to $M_s \gtrsim 2.5$ TeV while it becomes $M_s \gtrsim 1.1\,\text{TeV}$ for the extreme choice $g_s/4\pi = 1/128$. The later bound is to be compared with the limit obtained from dimension-eight operators that arise in the case of fermions from DD strings $M_s \gtrsim 0.63\,\text{TeV}$ [9, 24]. In the case of three sets of branes, the limits become $M_s \gtrsim 1.8$ and $0.81\,\text{TeV}$ for the two respective values of g_s.

A stronger bound can be obtained from the analysis of high precision low energy data in the presence of effective four-fermion operators that modify the μ-decay amplitude. They are obtained from the first term of eq. (6.2) with ψ and ψ' the muon and electron doublets, respectively, and with an overall negative sign. Using the results of ref. [26], we obtain $M_s \gtrsim 3.1\,\text{TeV}$ for $g_s \simeq 1/2$, or $M_s \gtrsim 2.8\,\text{TeV}$ for $g_s = g_2^2 \simeq 0.425$ with g_2 the SU(2) coupling at the weak scale. Note that in this case

JHEP05(2001)044

the ambiguity on the value of the coupling is not important. In the case of three sets of branes, the limits become $M_s \gtrsim 2.2\,\text{TeV}$.

In the case $\psi = \psi'$ as for Bhabha scattering in e^+e^- there is an additional contribution to the effective operator coming from the operators that are associated with the exchange of 77 states. In the case of two sets of branes, this leads to:

$$0.75\,\eta_{LL} = 0.75\,\eta_{RR} \simeq \eta_{LR} = \eta_{RL} = 1\,,$$
$$\Lambda \simeq \sqrt{\frac{4\pi}{0.59 g_s}} M_s\,, \tag{6.4}$$

where we see that the dominant effects are still due to the 33 channels.

Finally, for comparison, we would like to consider the case where Standard Model gauge interactions appear in a higher-dimensional D-brane and feel the presence of additional longitudinal dimensions. This can be obtained either by identifying observable gauge bosons with D7-brane states or with those of D4-branes obtained from the previous D3-branes by taking one of the transverse directions in the world-volume of D7-branes smaller than the string length and performing a T-duality. The resulting contact interaction is dominated by the effects due to exchange of KK excitations. These lead to an operator of the form (5.3). In the case of one transverse dimension, the sum over KK states gives [4]:

$$\Lambda_{\text{eff}}^{\text{KK}} \simeq -\frac{\pi^2}{3(1+\varepsilon)} R^2 g_s \sum_{a,b=L,R} \eta_{ab} \bar{\psi}_a \gamma^\mu \psi_a \bar{\psi}'_b \gamma_\mu \psi'_b\,. \tag{6.5}$$

Experimental constraints on such operators translate into lower bounds on the scale of compactification. For instance exchanges leading to vector interactions would lead to:

$$\eta_{LL} = \eta_{RR} = \eta_{LR} = \eta_{RL} = -1\,, \tag{6.6}$$
$$\Lambda \simeq \sqrt{\frac{12}{\pi g_s}} \frac{1}{R}\,. \tag{6.7}$$

Using LEP bounds [25] $\Lambda^-_{VV} \gtrsim 14\,\text{TeV}$ and $g_s = g^2_{\text{YM}} \sim 1/2$, with g_{YM} the gauge coupling, gives $R^{-1} \gtrsim 5.0\,\text{TeV}$. This bound becomes $R^{-1} \gtrsim 2.2\,\text{TeV}$ for the choice $g_s/4\pi = 1/128$ corresponding to contact terms dominated for instance by the exchange of KK excitations of photons. Low energy precision electroweak data lead instead to $R^{-1} \gtrsim 3.5\,\text{TeV}$ [12].

Acknowledgments

We thank Alessandro Strumia for useful comments on extracting experimental bounds using LEP1 data for charged currents. This work was supported in part by the European Commission under TMR contract ERBFMRX-CT96-0090 and RTN contract HPRN-CT-2000-00148, and in part by the INTAS contract 99-0590. The work of K.B. was also partially supported by NSERC of Canada and Fonds FCAR du Québec.

A. Four-scalar scattering amplitudes from ND open strings

The massless scalars arising from open strings with ND boundary conditions transform in spinorial representations of the SO(4) internal symmetry corresponding to the four twisted directions. The corresponding vertex operators are given (in the -1 ghost picture) by:

$$\mathcal{V}^{(a)}_{\rm ND}(x_a, k_a) = \sqrt{2g_s}\, l_s\, e^{ik_i\cdot X}\, \lambda_{j_a I_a}\, e^{-\phi}\, \eta^{(a)}_\alpha S^\alpha_{(4)} \prod_{j=1}^{4} \sigma^j_\pm(x_a)\,, \tag{A.1}$$

where the momenta k_a have components only in the directions longitudinal to the D3-brane and satisfy $k_a^2 = 0$. The $S^\alpha_{(4)}$ and $\eta^{(a)}_\alpha$ represent the spin field and the corresponding polarization in the internal SO(4), respectively. The computation of the ordered amplitude involving four scalars from ND open strings is straightforward and leads to:

$$\begin{aligned}\mathcal{A}(1_{j_1I_1}, 2_{j_2I_2}, 3_{j_3I_3}, 4_{j_4I_4}) = {}& -2ug_s\, l_s^2 \int_0^1 dx\, x^{-1-s\,l_s^2}(1-x)^{-1-t\,l_s^2}\, \frac{1}{[F(x)]^2}\,\times \\ & \times\Big[\bar\eta^{(1)}\Gamma_a\eta^{(2)}\bar\eta^{(4)}\Gamma^a\eta^{(3)}(1-x) + \bar\eta^{(1)}\Gamma_a\eta^{(4)}\bar\eta^{(2)}\Gamma^a\eta^{(3)}x\Big]\times \\ & \times\Bigg\{\frac{\delta_{I_1,\bar I_2}\delta_{I_3,\bar I_4}\delta_{\bar j_1,j_4}\delta_{j_2,\bar j_3}}{l_s^{-4}\prod_{i=1}^4 R_i}\sum_{m_i} e^{i2\pi\sum_i m_i a_i^{I_1I_3}}\, e^{-\pi\tau\sum_i \frac{m_i^2 l_s^2}{R_i^2}} + \\ & + \delta_{j_1,\bar j_2}\delta_{j_3,\bar j_4}\delta_{\bar I_1,I_4}\delta_{I_2,\bar I_3}\sum_{n_i} e^{-\pi\tau\sum_i(n_i+a_i^{I_1I_3})^2 R_i^2 l_s^{-2}}\Bigg\}\,,\end{aligned} \tag{A.2}$$

where n_i and m_i are integers. The total amplitude is obtained by summing up all orderings.

The low-energy limit $|s\,l_s^2| \ll 1$ and $|t\,l_s^2| \ll 1$ of the amplitude in (A.2) can be computed and is decomposed in two terms $\mathcal{A}_1^{\rm QFT}$ and $\mathcal{A}_2^{\rm QFT}$:

$$\begin{aligned}\mathcal{A}_1^{\rm QFT} = {}& -g_s\,\delta_{j_1,\bar j_2}\delta_{j_3,\bar j_4}\delta_{\bar I_1,I_4}\delta_{I_2,\bar I_3}\,\times \\ & \times\Bigg\{(s-u)\frac{\left[\bar\eta^{(1)}\Gamma_a\eta^{(4)}\bar\eta^{(2)}\Gamma^a\eta^{(3)}\right]}{l_s^{-4}\prod_{i=1}^4 R_i}\left[\sum_{m_i}\frac{e^{i2\pi\sum_i m_i a_i^{I_1I_3}}\,\delta^{-\sum_i \frac{m_i^2 l_s^2}{R_i^2}}}{t-\sum_i \frac{m_i^2}{R_i^2}}\right] + \\ & + (t-u)\left[\bar\eta^{(1)}\Gamma_a\eta^{(2)}\bar\eta^{(4)}\Gamma^a\eta^{(3)}\right]\times \\ & \times\left[\sum_{|n_i+a_i^{I_1I_3}|\,R_i<l_s}\frac{\delta^{-\sum_i(n_i+a_i^{I_1I_3})^2\,R_i^2\,l_s^{-2}}}{s-\sum_i(n_i+a_i^{I_1I_3})^2\,R_i^2\,l_s^{-4}}\right]\Bigg\} -\end{aligned}$$

$$
\begin{aligned}
&- g_s\, \delta_{I_1,\bar I_2}\delta_{I_3,\bar I_4}\delta_{\bar j_1,j_4}\delta_{j_2,\bar j_3} \times \\
&\times \left\{ (t-u)\frac{\left[\bar\eta^{(1)}\Gamma_a\eta^{(2)}\bar\eta^{(4)}\Gamma^a\eta^{(3)}\right]}{l_s^{-4}\prod_{i=1}^4 R_i} \left[\sum_{m_i} \frac{e^{i2\pi\sum_i m_i a_i^{I_1I_3}}\,\delta^{-\sum_i \frac{m_i^2 l_s^2}{R_i^2}}}{s-\sum_i \frac{m_i^2}{R_i^2}}\right] + \right.\\
&\quad + (s-u)\left[\bar\eta^{(1)}\Gamma_a\eta^{(4)}\bar\eta^{(2)}\Gamma^a\eta^{(3)}\right] \times \\
&\quad \left. \times \left[\sum_{|n_i+a_i^{I_1I_3}|\,R_i<l_s} \frac{\delta^{-\sum_i (n_i+a_i^{I_1I_3})^2\,R_i^2\,l_s^{-2}}}{t-\sum_i (n_i+a_i^{I_1I_3})^2\,R_i^2\,l_s^{-4}}\right]\right\}
\end{aligned}
\tag{A.3}
$$

and

$$
\begin{aligned}
\mathcal{A}_2^{\rm QFT} =& -g_s\,\delta_{j_1,\bar j_2}\delta_{j_3,\bar j_4}\delta_{\bar I_1,I_4}\delta_{I_2,\bar I_3} \times \\
&\times \left\{ \frac{\left[\bar\eta^{(1)}\Gamma_a\eta^{(4)}\bar\eta^{(2)}\Gamma^a\eta^{(3)}\right]}{l_s^{-4}\prod_{i=1}^4 R_i} \left[\sum_{m_i} e^{i2\pi\sum_i m_i a_i^{I_1I_3}}\,\delta^{-\sum_i \frac{m_i^2 l_s^2}{R_i^2}} \frac{t}{t-\sum_i \frac{m_i^2}{R_i^2}}\right] + \right.\\
&\quad + \left[\bar\eta^{(1)}\Gamma_a\eta^{(2)}\bar\eta^{(4)}\Gamma^a\eta^{(3)}\right] \times \\
&\quad \left. \times \left[\sum_{|n_i+a_i^{I_1I_3}|\,R_i<l_s} \delta^{-\sum_i (n_i+a_i^{I_1I_3})^2\,R_i^2 l_s^{-2}} \frac{s}{s-\frac{\sum_i (n_i+a_i^{I_1I_3})^2\,R_i^2}{l_s^4}}\right]\right\} - \\
&- g_s\,\delta_{I_1,\bar I_2}\delta_{I_3,\bar I_4}\delta_{\bar j_1,j_4}\delta_{j_2,\bar j_3} \times \\
&\times \left\{ \frac{\left[\bar\eta^{(1)}\Gamma_a\eta^{(2)}\bar\eta^{(4)}\Gamma^a\eta^{(3)}\right]}{l_s^{-4}\prod_{i=1}^4 R_i} \left[\sum_{m_i} e^{i2\pi\sum_i m_i a_i^{I_1I_3}}\,\delta^{-\sum_i \frac{m_i^2 l_s^2}{R_i^2}} \frac{s}{s-\sum_i \frac{m_i^2}{R_i^2}}\right] + \right.\\
&\quad + \left[\bar\eta^{(1)}\Gamma_a\eta^{(4)}\bar\eta^{(2)}\gamma^a\eta^{(3)}\right] \times \\
&\quad \left. \times \left[\sum_{|n_i+a_i^{I_1I_3}|\,R_i<l_s} \delta^{-\sum_i (n_i+a_i^{I_1I_3})^2\,R_i^2 l_s^{-2}} \frac{t}{t-\frac{\sum_i (n_i+a_i^{I_1I_3})^2\,R_i^2}{l_s^4}}\right]\right\}
\end{aligned}
\tag{A.4}
$$

which reproduce the effective field theory contributions from the exchange of gauge bosons and their KK excitations. The result in eq. (A.3) arises when the derivative, present in the interaction vertex, acts on the external ND scalar states. Taking $a_i^I = 0$, eq. (A.4) contains a sum of terms obtained when acting by the derivative on the internal massive KK excitation of 77 states. In addition, it contains a four-scalar contact term that corresponds to $m_i = n_i = 0$

$$
\begin{aligned}
\mathcal{A}^{\rm contact} =& -g_s\,\delta_{j_1,\bar j_2}\delta_{j_3,\bar j_4}\delta_{\bar I_1,I_4}\delta_{I_2,\bar I_3}\left\{\frac{\bar\eta^{(1)}\Gamma_a\eta^{(4)}\bar\eta^{(2)}\Gamma^a\eta^{(3)}}{l_s^{-4}\prod_{i=1}^4 R_i} + \bar\eta^{(1)}\Gamma_a\eta^{(2)}\bar\eta^{(4)}\Gamma^a\eta^{(3)}\right\} - \\
&- g_s\delta_{I_1,\bar I_2}\delta_{I_3,\bar I_4}\delta_{\bar j_1,j_4}\delta_{j_2,\bar j_3}\left\{\frac{\bar\eta^{(1)}\Gamma_a\eta^{(2)}\bar\eta^{(4)}\Gamma^a\eta^{(3)}}{l_s^{-4}\prod_{i=1}^4 R_i} + \bar\eta^{(1)}\Gamma_a\eta^{(4)}\bar\eta^{(2)}\gamma^a\eta^{(3)}\right\}
\end{aligned}
\tag{A.5}
$$

which reproduces the usual quartic coupling from D-terms in supersymmetric theories. Dimension-six operators involving four saclars can be extracted easily from the above results and they are subleading.

References

[1] for reviews see, for example, J. Polchinski, *String theory*, vols. I, II, Cambridge University Press, 1998; *Tasi lectures on D-branes*, hep-th/9611050;
M.J. Duff, *M-theory (the theory formerly known as strings)*, *Int. J. Mod. Phys.* **A 11** (1996) 5623;
C.P. Bachas, *Lectures on D-branes*, hep-th/9806199;
C.V. Johnson, *D-brane primer*, hep-th/0007170.

[2] A. Sagnotti, in Cargese 87, *Strings on orbifolds*, G. Mack et al. eds., Pergamon Press, 1988 p. 521;
P. Hořava, *Strings on world sheet orbifolds*, *Nucl. Phys.* **B 327** (1989) 461;
J. Dai, R. Leigh and J. Polchinski, *New connections between string theories*, *Mod. Phys. Lett.* **A 4** (1989) 2073;
G. Pradisi and A. Sagnotti, *Open string orbifolds*, *Phys. Lett.* **B 216** (1989) 59;
M. Bianchi and A. Sagnotti, *On the systematics of open string theories*, *Phys. Lett.* **B 247** (1990) 517.

[3] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Y.S. Stanev, *Chiral asymmetry in four-dimensional open-string vacua*, *Phys. Lett.* **B 385** (1996) 96 [hep-th/9606169];
G. Aldazabal, A. Font, L.E. Ibáñez and G. Violero, *$D = 4$, $N = 1$, type-IIB orientifolds*, *Nucl. Phys.* **B 536** (1998) 29 [hep-th/9804026];
G. Aldazabal, L.E. Ibáñez, F. Quevedo and A.M. Uranga, *D-Branes at singularities: a bottom-up approach to the string embedding of the standard model*, *J. High Energy Phys.* **08** (2000) 002 [hep-th/0005067];
G. Aldazabal, L.E. Ibáñez and F. Quevedo, *A D-brane alternative to the MSSM*, *J. High Energy Phys.* **02** (2000) 015 [hep-ph/0001083].

[4] I. Antoniadis and K. Benakli, *Limits on extra dimensions in orbifold compactifications of superstrings*, *Phys. Lett.* **B 326** (1994) 69 [hep-th/9310151].

[5] E. Accomando, I. Antoniadis and K. Benakli, *Looking for tev-scale strings and extra-dimensions*, *Nucl. Phys.* **B 579** (2000) 3 [hep-ph/9912287].

[6] I. Antoniadis, *A possible new dimension at a few TeV*, *Phys. Lett.* **B 246** (1990) 377;
E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, *Nucl. Phys.* **B 471** (1996) 135 [hep-th/9602070];
J.D. Lykken, *Weak scale superstrings*, *Phys. Rev.* **D 54** (1996) 3693 [hep-th/9603133].

[7] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315];

I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *New dimensions at a millimeter to a fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398].

[8] See also, for example, K.R. Dienes, E. Dudas and T. Gherghetta, *Extra spacetime dimensions and unification*, *Phys. Lett.* **B 436** (1998) 55 [hep-ph/9803466]; *Grand unification at intermediate mass scales through extra dimensions*, *Nucl. Phys.* **B 537** (1999) 47 [hep-ph/9806292];
G. Shiu and S.H.H. Tye, *Tev scale superstring and extra dimensions*, *Phys. Rev.* **D 58** (1998) 106007 [hep-th/9805157];
A. Pomarol and M. Quirós, *The standard model from extra dimensions*, *Phys. Lett.* **B 438** (1998) 255 [hep-ph/9806263];
C. Bachas, *Unification with low string scale*, *J. High Energy Phys.* **11** (1998) 023 [hep-ph/9807415];
D. Ghilencea and G.G. Ross, *Unification and extra space-time dimensions*, *Phys. Lett.* **B 442** (1998) 165 [hep-ph/9809217];
Z. Kakushadze and S.H.H. Tye, *Brane world*, *Nucl. Phys.* **B 548** (1999) 180 [hep-th/9809147];
K. Benakli, *Phenomenology of low quantum gravity scale models*, *Phys. Rev.* **D 60** (1999) 104002 [hep-ph/9809582];
C.P. Burgess, L.E. Ibáñez and F. Quevedo, *Strings at the intermediate scale or is the fermi scale dual to the Planck scale?*, *Phys. Lett.* **B 447** (1999) 257 [hep-ph/9810535];
G.F. Giudice, R. Rattazzi and J.D. Wells, *Quantum gravity and extra dimensions at high-energy colliders*, *Nucl. Phys.* **B 544** (1999) 3 [hep-ph/9811291];
E.A. Mirabelli, M. Perelstein and M.E. Peskin, *Collider signatures of new large space dimensions*, *Phys. Rev. Lett.* **82** (1999) 2236 [hep-ph/9811337];
T. Han, J.D. Lykken and R.-J. Zhang, *On Kaluza-Klein states from large extra dimensions*, *Phys. Rev.* **D 59** (1999) 105006 [hep-ph/9811350];
J.L. Hewett, *Indirect collider signals for extra dimensions*, *Phys. Rev. Lett.* **82** (1999) 4765 [hep-ph/9811356];
L.E. Ibáñez, C. Munoz and S. Rigolin, *Aspects of type-I string phenomenology*, *Nucl. Phys.* **B 553** (1999) 43 [hep-ph/9812397];
I. Antoniadis and B. Pioline, *Low-scale closed strings and their duals*, *Nucl. Phys.* **B 550** (1999) 41 [hep-th/9902055];
L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064];
M. Bando, T. Kugo, T. Noguchi and K. Yoshioka, *Brane fluctuation and suppression of Kaluza-Klein mode couplings*, *Phys. Rev. Lett.* **83** (1999) 3601 [hep-ph/9906549];
K. Benakli and Y. Oz, *Small instantons and weak scale string theory*, *Phys. Lett.* **B 472** (2000) 83 [hep-th/9910090];
E. Dudas and J. Mourad, *String theory predictions for future accelerators*, *Nucl. Phys.* **B 575** (2000) 3 [hep-th/9911019];
N. Kaloper, J. March-Russell, G.D. Starkman and M. Trodden, *Compact hyperbolic extra dimensions: branes, Kaluza-Klein modes and cosmology*, *Phys. Rev. Lett.* **85**

(2000) 928 [hep-ph/0002001];
J.F. Gunion and B. Grzadkowski, *Extra dimension Kaluza-Klein excitations and electroweak symmetry breaking*, hep-ph/0004058;
I. Antoniadis, K. Benakli and M. Quirós, *Radiative symmetry breaking in brane models*, *Nucl. Phys.* **B 583** (2000) 35 [hep-ph/0004091];
H. Georgi, A.K. Grant and G. Hailu, *Chiral fermions, orbifolds, scalars and fat branes*, *Phys. Rev.* **D 63** (2001) 064027 [hep-ph/0007350];
R. Barbieri, L.J. Hall and Y. Nomura, *A constrained standard model from a compact extra dimension*, *Phys. Rev.* **D 63** (2001) 105007 [hep-ph/0011311];
For a review and more references see I. Antoniadis and K. Benakli, *Large dimensions and string physics in future colliders*, *Int. J. Mod. Phys.* **A 15** (2000) 4237 [hep-ph/0007226].

[9] S. Cullen, M. Perelstein and M.E. Peskin, *TeV strings and collider probes of large extra dimensions*, *Phys. Rev.* **D 62** (2000) 055012 [hep-ph/0001166].

[10] I. Antoniadis, E. Kiritsis and T.N. Tomaras, *A D-brane alternative to unification*, *Phys. Lett.* **B 486** (2000) 186 [hep-ph/0004214].

[11] I. Antoniadis, K. Benakli and M. Quirós, *Production of Kaluza-Klein states at future colliders*, *Phys. Lett.* **B 331** (1994) 313 [hep-ph/9403290]; *Direct collider signatures of large extra dimensions*, *Phys. Lett.* **B 460** (1999) 176 [hep-ph/9905311];
A. Donini and S. Rigolin, *Anisotropic type-I string compactification, winding modes and large extra dimensions*, *Nucl. Phys.* **B 550** (1999) 59 [hep-ph/9901443];
P. Nath, Y. Yamada and M. Yamaguchi, *Probing the nature of compactification with Kaluza-Klein excitations at the large hadron collider*, *Phys. Lett.* **B 466** (1999) 100 [hep-ph/9905415];
A.D. Rujula, A. Donini, M.B. Gavela and S. Rigolin, *Fat brane phenomena*, *Phys. Lett.* **B 482** (2000) 195 [hep-ph/0001335].

[12] P. Nath and M. Yamaguchi, *Effects of extra space-time dimensions on the fermi constant*, *Phys. Rev.* **D 60** (1999) 116004 [hep-ph/9902323]; *Effects of Kaluza-Klein excitations on $g_\mu - 2$*, *Phys. Rev.* **D 60** (1999) 116006 [hep-ph/9903298];
M. Masip and A. Pomarol, *Effects of SM Kaluza-Klein excitations on electroweak observables*, *Phys. Rev.* **D 60** (1999) 096005 [hep-ph/9902467];
W.J. Marciano, *Fermi constants and 'new physics'*, *Phys. Rev.* **D 60** (1999) 093006 [hep-ph/9903451];
A. Strumia, *Bounds on Kaluza-Klein excitations of the SM vector bosons from electroweak tests*, *Phys. Lett.* **B 466** (1999) 107 [hep-ph/9906266];
T.G. Rizzo and J.D. Wells, *Electroweak precision measurements and collider probes of the standard model with large extra dimensions*, *Phys. Rev.* **D 61** (2000) 016007 [hep-ph/9906234];
R. Casalbuoni, S.D. Curtis, D. Dominici and R. Gatto, *SM Kaluza-Klein excitations and electroweak precision tests*, *Phys. Lett.* **B 462** (1999) 48 [hep-ph/9907355];
C.D. Carone, *Electroweak constraints on extended models with extra dimensions*, *Phys.*

Rev. **D 61** (2000) 015008 [hep-ph/9907362];
A. Delgado, A. Pomarol and M. Quirós, *Electroweak and flavor physics in extensions of the standard model with large extra dimensions*, *J. High Energy Phys.* **01** (2000) 030 [hep-ph/9911252];
M. Masip, *Ultraviolet dependence of Kaluza-Klein effects on electroweak observables*, *Phys. Rev.* **D 62** (2000) 105012 [hep-ph/0007048].

[13] I. Antoniadis, E. Dudas and A. Sagnotti, *Brane supersymmetry breaking*, *Phys. Lett.* **B 464** (1999) 38 [hep-th/9908023];
G. Aldazabal and A.M. Uranga, *Tachyon-free non-supersymmetric type-IIB orientifolds via brane-antibrane systems*, *J. High Energy Phys.* **10** (1999) 024 [hep-th/9908072];
C. Angelantonj, I. Antoniadis, G. D'Appollonio, E. Dudas and A. Sagnotti, *Type-I vacua with brane supersymmetry breaking*, *Nucl. Phys.* **B 572** (2000) 36 [hep-th/9911081].

[14] M.R. Garousi and R.C. Myers, *Superstring scattering from D-branes*, *Nucl. Phys.* **B 475** (1996) 193 [hep-th/9603194];
A. Hashimoto and I.R. Klebanov, *Decay of excited D-branes*, *Phys. Lett.* **B 381** (1996) 437 [hep-th/9604065].

[15] D. Friedan, E. Martinec and S. Shenker, *Conformal invariance, supersymmetry and string theory*, *Nucl. Phys.* **B 271** (1986) 93;
V.A. Kostelecky, O. Lechtenfeld, W. Lerche, S. Samuel and S. Watamura; *Nucl. Phys.* **B 288** (1987) 173.

[16] G. Veneziano, *Construction of a crossing-symmetric, Regge behaved amplitude for linearly rising trajectories*, *Nuovo Cim.* **A57** (1968) 190;
J.H. Schwarz, *Superstring theory*, *Phys. Rep.* **89** (1982) 223.

[17] L. Dixon, D. Friedan, E. Martinec and S. Shenker, *The conformal field theory of orbifolds*, *Nucl. Phys.* **B 282** (1987) 13;
S. Hamidi and C. Vafa, *Interactions on orbifolds*, *Nucl. Phys.* **B 279** (1987) 465.

[18] E. Gava, K.S. Narain and M.H. Sarmadi, *On the bound states of p- and $(p+2)$-branes*, *Nucl. Phys.* **B 504** (1997) 214 [hep-th/9704006].

[19] D. Mumford, *Tata lectures on* Θ, Birkhauser, 1983.

[20] N. Moeller, A. Sen and B. Zwiebach, *J. High Energy Phys.* **08** (2000) 039.

[21] A. Hashimoto, *Dynamics of Dirichlet-Neumann open strings on D-branes*, *Nucl. Phys.* **B 496** (1997) 243 [hep-th/9608127];
B. Chen, H. Itoyama, T. Matsuo and K. Murakami, *pp' system with B-field, branes at angles and noncommutative geometry*, *Nucl. Phys.* **B 576** (2000) 177 [hep-th/9910263];
J. Fröhlich, O. Grandjean, A. Recknagel and V. Schomerus, *Fundamental strings in Dp-Dq brane systems*, *Nucl. Phys.* **B 583** (2000) 381 [hep-th/9912079].

[22] A. Sagnotti, *A note on the Green-Schwarz mechanism in open string theories*, *Phys. Lett.* **B 294** (1992) 196 [hep-th/9210127];
L.E. Ibáñez, R. Rabadán and A.M. Uranga, *Anomalous* U(1)*'s in type-I and type-IIB* $D = 4$, $N = 1$ *string vacua*, *Nucl. Phys.* **B 542** (1999) 112 [hep-th/9808139];
E. Poppitz, *On the one loop Fayet-Iliopoulos term in chiral four dimensional type-I orbifolds*, *Nucl. Phys.* **B 542** (1999) 31 [hep-th/9810010];
M. Cvetič, L.L. Everett, P. Langacker and J. Wang, *Blowing-up the four-dimensional* Z_3 *orientifold*, *J. High Energy Phys.* **04** (1999) 020 [hep-th/9903051];
Z. Lalak, S. Lavignac and H.P. Nilles, *String dualities in the presence of anomalous* U(1) *symmetries*, *Nucl. Phys.* **B 559** (1999) 48 [hep-th/9903160].

[23] E. Eichten, K. Lane and M.E. Peskin, *New tests for quark and lepton substructure*, *Phys. Rev. Lett.* **50** (1983) 811.

[24] D. Bourilkov, *Search for tev strings and new phenomena in Bhabha scattering at LEP2*, *Phys. Rev.* **D 62** (2000) 076005 [hep-ph/0002172];
L3 collaboration, *Search for manifestations of new physics in fermion pair production at LEP*, *Phys. Lett.* **B 489** (2000) 81 [hep-ex/0005028].

[25] LEP and SLD experiments, preprint CERN-EP-2000-016.

[26] R. Barbieri and A. Strumia, *What is the limit on the Higgs mass?*, *Phys. Lett.* **B 462** (1999) 144 [hep-ph/9905281].